Mathematics

Part - I

STANDARD TEN

$$Y = 2$$

$$(0,3)$$

$$(4,0)$$

$$3x + 4y = 12$$

$1 + 2 + 3 + \cdots \qquad \cdots + 78 + 79 + 80$

$= (1 + 80) + (2 + 79) + \cdots + (39 + 42) + (40 + 41)$

Mathematics

Part I

STANDARD TEN

Maharashtra State Bureau of Textbook Production and Curriculum Research, Pune - 411 004

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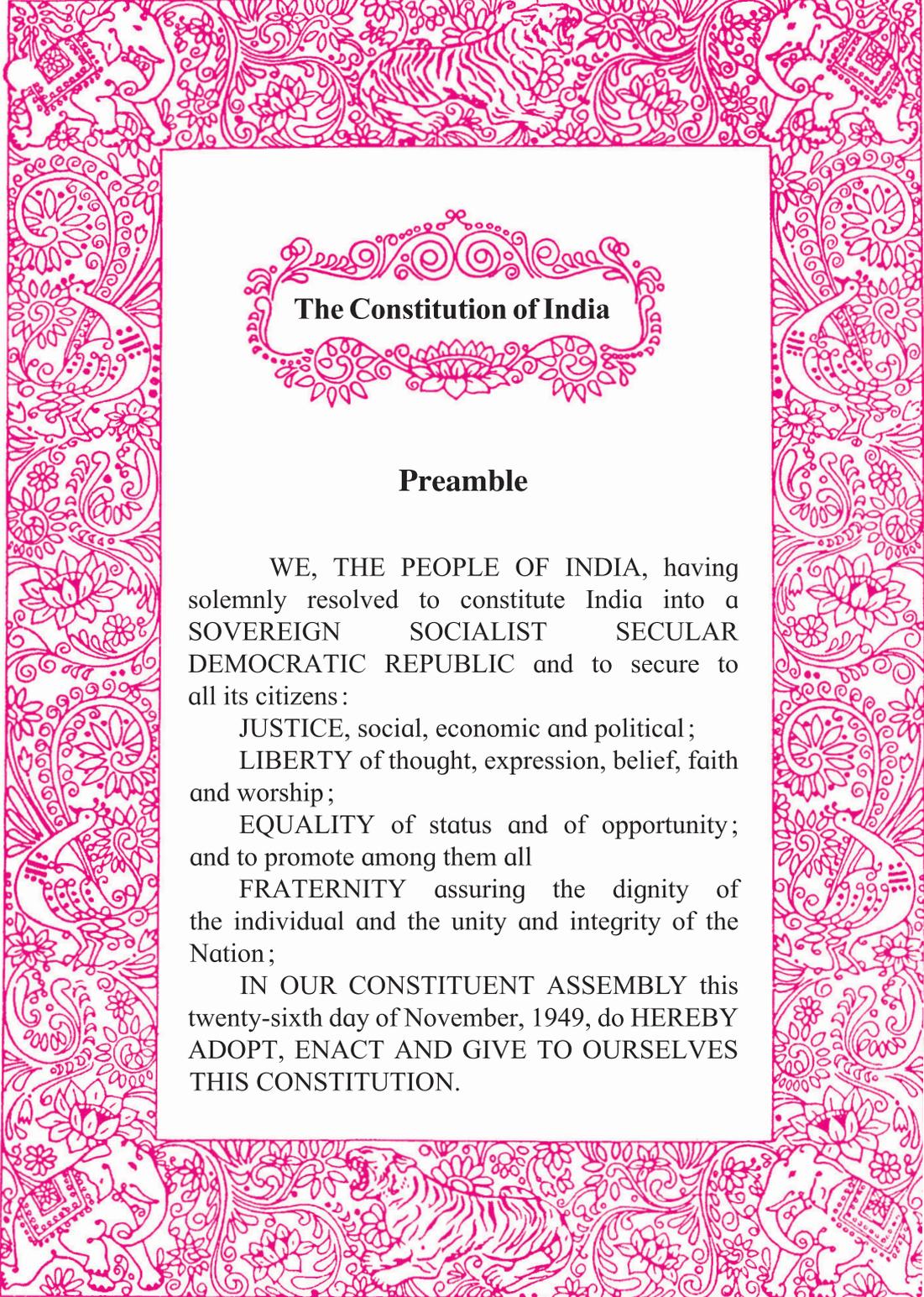
The Constitution of India

Preamble

WE, THE PEOPLE OF INDIA, having solemnly resolved to constitute India into a SOVEREIGN SOCIALIST SECULAR DEMOCRATIC REPUBLIC and to secure to all its citizens:

JUSTICE, social, economic and political;

LIBERTY of thought, expression, belief, faith and worship;

EQUALITY of status and of opportunity; and to promote among them all

FRATERNITY assuring the dignity of the individual and the unity and integrity of the Nation;

IN OUR CONSTITUENT ASSEMBLY this twenty-sixth day of November, 1949, do HEREBY ADOPT, ENACT AND GIVE TO OURSELVES THIS CONSTITUTION.

NATIONAL ANTHEM

Jana-gana-mana-adhināyaka jaya hē
     Bhārata-bhāgya-vidhātā,

Panjāba-Sindhu-Gujarāta-Marāthā
     Drāvida-Utkala-Banga

Vindhya-Himāchala-Yamunā-Gangā
     uchchala-jaladhi-taranga

Tava subha nāmē jāgē, tava subha āsisa māgē,
     gāhē tava jaya-gāthā,

Jana-gana-mangala-dāyaka jaya hē
     Bhārata-bhāgya-vidhātā,

Jaya hē, Jaya hē, Jaya hē,
     Jaya jaya jaya, jaya hē.

PLEDGE

India is my country. All Indians are my brothers and sisters.

I love my country, and I am proud of its rich and varied heritage. I shall always strive to be worthy of it.

I shall give my parents, teachers and all elders respect, and treat everyone with courtesy.

To my country and my people, I pledge my devotion. In their well-being and prosperity alone lies my happiness.

Preface

Dear Students,

Welcome to the tenth standard!

This year you will study two text books - Mathematics Part-I and Mathematics Part-II

The main areas in the book Mathematics Part-I are Algebra, Graph, Financial planning and Statistics. All of these topics were introduced in the ninth standard. This year you will study some more details of the same. The new tax system, GST is introduced in Financial planning. Wherever a new unit, formula or application is introduced, its lucid explanation is given. Each chapter contains illustrative solved examples and sets of questions for practice. In addition, some questions in practice sets are star-marked, indicating that they are challenging for talented students. After tenth standard, some students do not opt for Mathematics. This book gives them basic concepts and mathematics needed to work in other fields. The matter under the head 'For more information' is useful for those students who wish to study mathematics after tenth standard and achieve proficiency in it. So they are earnestly advised to study it. Read the book thoroughly at least once and understand the concepts.

Additional useful audio-visual material regarding each lesson will be available to you by using Q.R. code through 'App'. It will definitely be useful to you for your studies.

Much importance is given to the tenth standard examination. You are advised not to take the stress and study to the best of your ability to achieve expected success. Best wishes for it !

(Dr. Sunil Magar)
Director

Pune
Date : 18 March 2018, Gudhipadva
Indian Solar Year : 27 Falgun 1939

Maharashtra State Bureau of Textbook
Production and Curriculum Research, Pune.

It is expected that students will develop the following competencies after studying Mathematics– Part I syllabus in standard X

Area	Topic	Competency Statements
1. Knowledge of numbers	1.1 Arithmetic Progression	The students will be able to- • solve examples using Arithmetic Progression. • plan steps to achieve a goal in future.
2. Algebra	2.1 Quadratic Equations 2.2 Linear equations in two variables	• solve day to day problems which can be expressed in the form of quadratic equations. • decide the number of variables required to find solutions of word problems. • convert a word problem into an equation in two variables and find its solution.
3. Commercial Mathematics	3.1 Financial planning	• understand the concepts of savings and investments. • get familiar with financial transactions in business, profession etc.
4. Statistics and Probability	4.1 Probability 4.2 Graph and measures of central tendencies	• use the concept of probaility in games, voting etc. • present the collected data in the form of graphs or pictures deciding the suitable form of presentation. • find the mean, median and mode of a provided classified data.

Instructions for Teachers

Read the book in detail and understand the content thoroughly.Take the help of activities to explain different topics, to verify the formulae etc.

Practicals is also a means of evaluation. Activities given can also be used for this purpose. Encourage the students to think independently.Compliment a student if he solves an example by a different and logically correct method.

List of some practicals (Specimen)

1. On a graph paper, draw a line parallel to the X-axis or Y-axis. Write coordinates of any four points on the line. Write how the equation of the line can be obtained from the coordinates.

   [Instead of parallel lines, lines passing through the origin or intersecting the X or Y-axis can also be considered]

2. Bear a two-digit number in mind. Without disclosing it, cunstruct a puzzle. Create two algebraic relations between the two digits of the number and solve the puzzle.

   [The above practical can be extended to a three-digit number also.]

3. Read the information about contents on a food packet. Show the information by a pie diagram. For example, see the chart of contents like carbohydrates, proteins, vitamines etc. per given weight on a buiscuit packet. Show the proportion of the contents by a pie diagram. The contents can be divided into four classes as carbohydrates, proteins, fats and others.

4. Prepare a frequncy distribution table given by the teacher in Excel sheet on a computer. From the table draw a frequency polygon and a histogram in Excel.

5. Roll a die ten times and record the outcomes in the form of a table.

6. Observe the tax invoice given by your teacher. Record all of its contents. Re-calculate the taxes and verify their correctness.

7. Calculate the sum of first n natural numbers given by your teacher through the following activity. For example to find the sum of first four natural numbers, take a square-grid piece of paper of 4×5 squares. Then cut it as shown in the figure. Hence verify the formula $S_n = \dfrac{n(n+1)}{2}$ (Here $n = 4$)
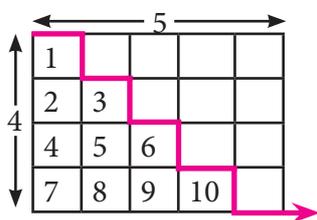
   $$S_n = \frac{n(n+1)}{2} \quad \therefore S_4 = \frac{4\,(4+1)}{2} = \frac{4\times 5}{2} = \frac{20}{2} = 10$$

   [Note: Here $a = 1$ and $d = 1$. The activity can be done taking different values of a and d. Similarly, you can find the sum of even or odd numbers, cubes of natural numbers etc.]

8. Write $\alpha = 6$ on one side of a card sheet and $\alpha = -6$ on its backside. Similarly, write $\beta = -3$ on one side of another card sheet and $\beta = 7$ on its backside. From these values, form different values of $(\alpha + \beta)$ and $(\alpha\beta)$; using these values form quadratic equations.

INDEX

Chapters	Pages

1 ▶ Linear Equations in Two Variables

- Methods of solving linear equations in two variables – graphical method, Cramer's method
- Equations that can be transformed in linear equation in two variables
- Application of simultaneous equations

Let's recall.

Linear equation in two variables

An equation which contains two variables and the degree of each term containing variable is one, is called a linear equation in two variables.

$ax + by + c = 0$ is the general form of a linear equation in two variables; a, b, c are real numbers and a, b are not equal to zero at the same time.

Ex. $3x - 4y + 12 = 0$ is the general form of equation $3x = 4y - 12$

Activity : Complete the following table

No.	Equation	Is the equation a linear equation in 2 variables ?
1	$4m + 3n = 12$	Yes
2	$3x^2 - 7y = 13$	
3	$\sqrt{2}\,x - \sqrt{5}\,y = 16$	
4	$0x + 6y - 3 = 0$	
5	$0.3x + 0y - 36 = 0$	
6	$\dfrac{4}{x} + \dfrac{5}{y} = 4$	
7	$4xy - 5y - 8 = 0$	

Simultaneous linear equations

When we think about two linear equations in two variables at the same time, they are called simultaneous equations.

Last year we learnt to solve simultaneous equations by eliminating one variable. Let us revise it.

Ex. **(1)** Solve the following simultaneous equations.

(1) $5x - 3y = 8$; $3x + y = 2$

Solution :

Method I : $5x - 3y = 8$ . . . (I)

$3x + y = 2$ . . . (II)

Multiplying both sides of equation (II) by 3.

$9x + 3y = 6$ . . . (III)

$5x - 3y = 8$ . . . (I)

Now let us add equations (I) and (III)

$5x - 3y = 8$

$+ \ 9x + 3y = 6$

$\overline{ 14x = 14}$

$\therefore x = 1$

substituting $x = 1$ in equation (II)

$3x + y = 2$

$\therefore 3 \times 1 + y = 2$

$\therefore 3 + y = 2$

$\therefore \qquad y = -1$

solution is $x = 1$, $y = -1$; it is also written as $(x, y) = (1, -1)$

Method (II)

$5x - 3y = 8$ . . . (I)

$3x + y = 2$ . . . (II)

Let us write value of y in terms of x from equation (II) as

$y = 2 - 3x$ . . . (III)

Substituting this value of y in equation (I).

$5x - 3y = 8$

$\therefore 5x - 3(2 - 3x) = 8$

$\therefore 5x - 6 + 9x = 8$

$\therefore 14x - 6 = 8$

$\therefore 14x = 8 + 6$

$\therefore 14x = 14$

$\therefore \qquad x = 1$

Substituting $x = 1$ in equation (III).

$y = 2 - 3x$

$\therefore y = 2 - 3 \times 1$

$\therefore y = 2 - 3$

$\therefore y = -1$

$x = 1$, $y = -1$ is the solution.

Ex. (2) Solve : $3x + 2y = 29$; $5x - y = 18$

Solution : $3x + 2y = 29$. . . (I) and $5x - y = 18$ . . . (II)

Let's solve the equations by eliminating 'y'. Fill suitably the boxes below.

Multiplying equation (II) by 2.

$\therefore\ 5x \times \boxed{} - y \times \boxed{} = 18 \times \boxed{}$

$\therefore\ 10x - 2y = \boxed{}$ . . . (III)

Let's add equations (I) and (III)

$3x + 2y = 29$

$+ \boxed{} - \boxed{} = \boxed{}$

$\boxed{} = \boxed{}$ $\qquad \therefore\ x = \boxed{}$

Substituting $x = 5$ in equation (I)

$3x + 2y = 29$

$\therefore\ 3 \times \boxed{} + 2y = 29$

$\therefore\ \boxed{} + 2y = 29$

$\therefore\ \qquad 2y = 29 - \boxed{}$

$\therefore\ \qquad 2y = \boxed{}$ $\qquad \therefore\ y = \boxed{}$

$(x, y) = (\boxed{}, \boxed{})$ is the solution.

Ex. (3) Solve : $15x + 17y = 21$; $17x + 15y = 11$

Solution : $15x + 17y = 21$. . . (I)

$17x + 15y = 11$ . . . (II)

In the two equations above, the coefficients of x and y are interchanged. While solving such equations we get two simple equations by adding and subtracting the given equations. After solving these equations, we can easily find the solution.

Let's add the two given equations.

$15x + 17y = 21$

$+ 17x + 15y = 11$

$32x + 32y = 32$

Dividing both sides of the equation by 32.

$$x + y = 1 \ldots \text{(III)}$$

Now, let's subtract equation (II) from (I)

$$\begin{array}{r} 15x + 17y = 21 \\ - \quad 17x \pm 15y = \underline{\pm 11} \\ \hline -2x + 2y = 10 \end{array}$$

dividing the equation by 2.

$$-x + y = 5 \ldots \text{(IV)}$$

Now let's add equations (III) and (V).

$$\begin{array}{r} x + y = 1 \\ + \quad -x + y = 5 \\ \hline \therefore \quad 2y = 6 \qquad \therefore \quad y = 3 \end{array}$$

Place this value in equation (III).

$$x + y = 1$$

$$\therefore x + 3 = 1$$

$$\therefore x = 1 - 3 \qquad \therefore x = -2$$

$(x, y) = (-2, 3)$ is the solution.

Practice Set 1.1

(1) Complete the following activity to solve the simultaneous equations.

$$5x + 3y = 9 \ \text{-----}\text{(I)}$$

$$2x - 3y = 12 \ \text{-----} \ \text{(II)}$$

Let's add eqations (I) and (II).

$$\begin{array}{r} 5x + 3y = 9 \\ + \quad 2x - 3y = 12 \\ \hline \boxed{}\ x = \boxed{} \\ x = \dfrac{\boxed{}}{\boxed{}} \qquad x = \boxed{} \end{array}$$

Place $x = 3$ in equation (I).

$$5 \times \boxed{} + 3y = 9$$

$$3y = 9 - \boxed{}$$

$$3y = \boxed{}$$

$$y = \dfrac{\boxed{}}{3}$$

$$y = \boxed{}$$

$\therefore$ Solution is $(x, y) = (\boxed{}, \boxed{})$.

2. Solve the following simultaneous equations.

    (1) $3a + 5b = 26$; $a + 5b = 22$ (2) $x + 7y = 10$; $3x - 2y = 7$

    (3) $2x - 3y = 9$; $2x + y = 13$ (4) $5m - 3n = 19$; $m - 6n = -7$

    (5) $5x + 2y = -3$; $x + 5y = 4$ (6) $\dfrac{1}{3}x + y = \dfrac{10}{3}$; $2x + \dfrac{1}{4}y = \dfrac{11}{4}$

    (7) $99x + 101y = 499$; $101x + 99y = 501$

    (8) $49x - 57y = 172$; $57x - 49y = 252$

Let's recall.
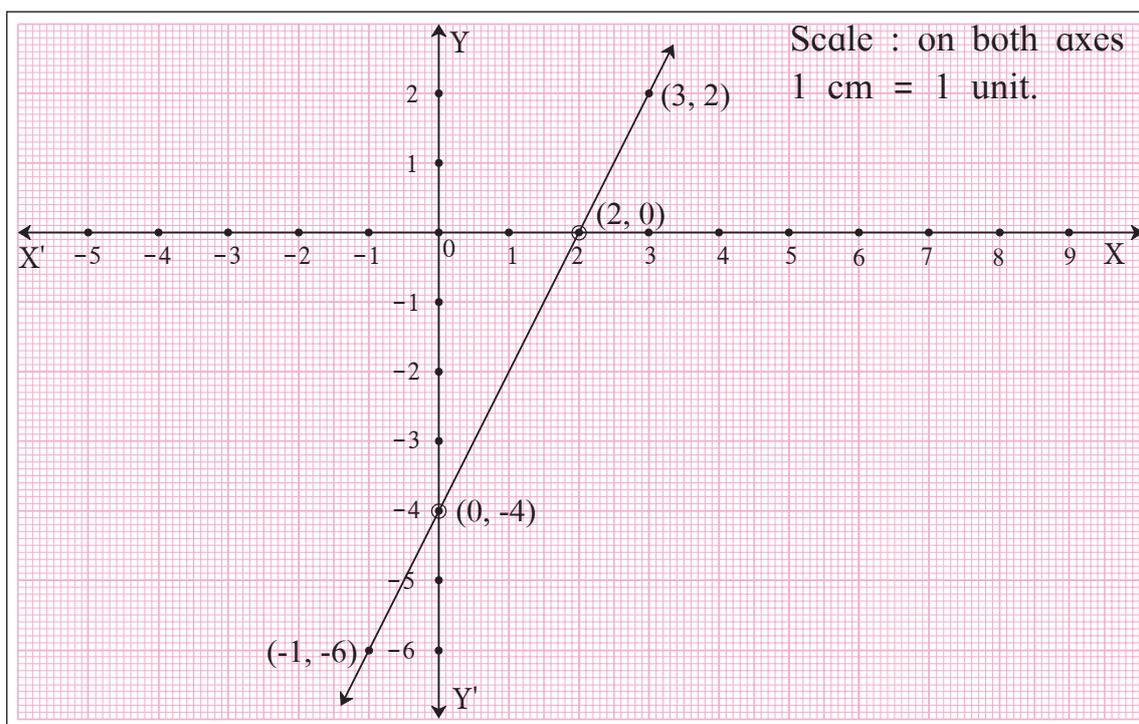
Graph of a linear equation in two variables

    In the 9th standard we learnt that the graph of a linear equation in two variables is a straight line. The ordered pair which satisfies the equation is a solution of that equation. The ordered pair represents a point on that line.

Ex.   Draw graph of $2x - y = 4$.

Solution : To draw a graph of the equation let's write 4 ordered pairs.

x	0	2	3	-1
y	-4	0	2	-6
(x, y)	$(0, -4)$	$(2, 0)$	$(3, 2)$	$(-1, -6)$

To obtain ordered pair by simple way let's take $x = 0$ and then $y = 0$.

Scale : on both axes
1 cm = 1 unit.

Steps to follow for drawing a graph of linear equation in two variables.

Find at least 4 ordered pairs for given equation

↓

Draw X- axis, Y-axis on graph paper and plot the points

↓

See that all 4 points lie on one line
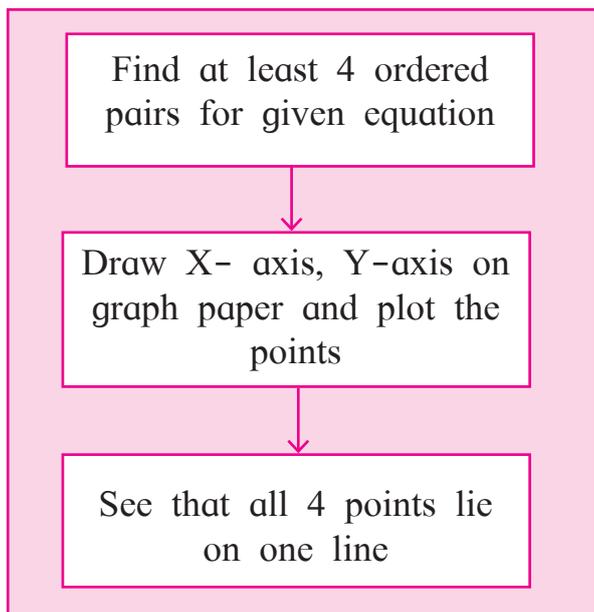
Two points are sufficient to represent a line, but if co-ordinates of one of the two points are wrong then you will not get a correct line.

If you plot three points and if they are non collinear then it is understood that one of the points is wrongly plotted. But it is not easy to identify the incorrect point.

If we plot four points, it is almost certain that three of them will be collinear.

A linear equation $y = 2$ is also written as $0x + y = 2$. The graph of this line is parallel to X- axis; as for any value of x, y is always 2.

x	1	4	−3
y	2	2	2
(x, y)	$(1, 2)$	$(4, 2)$	$(−3, 2)$

Similarly equation $x = 2$ is written as $x + 0y = 2$ and its graph is parallel to Y-axis.
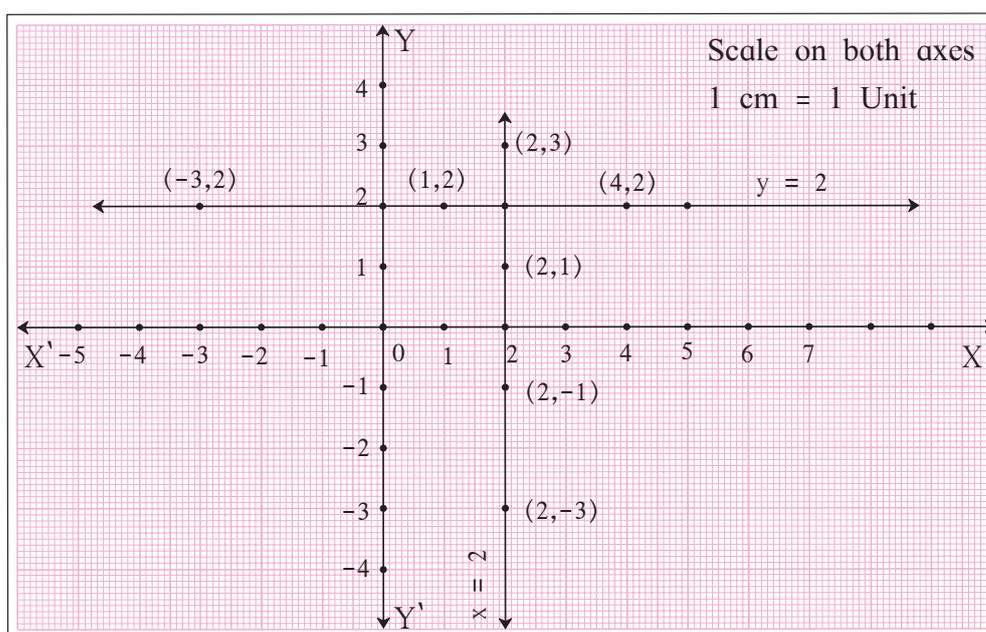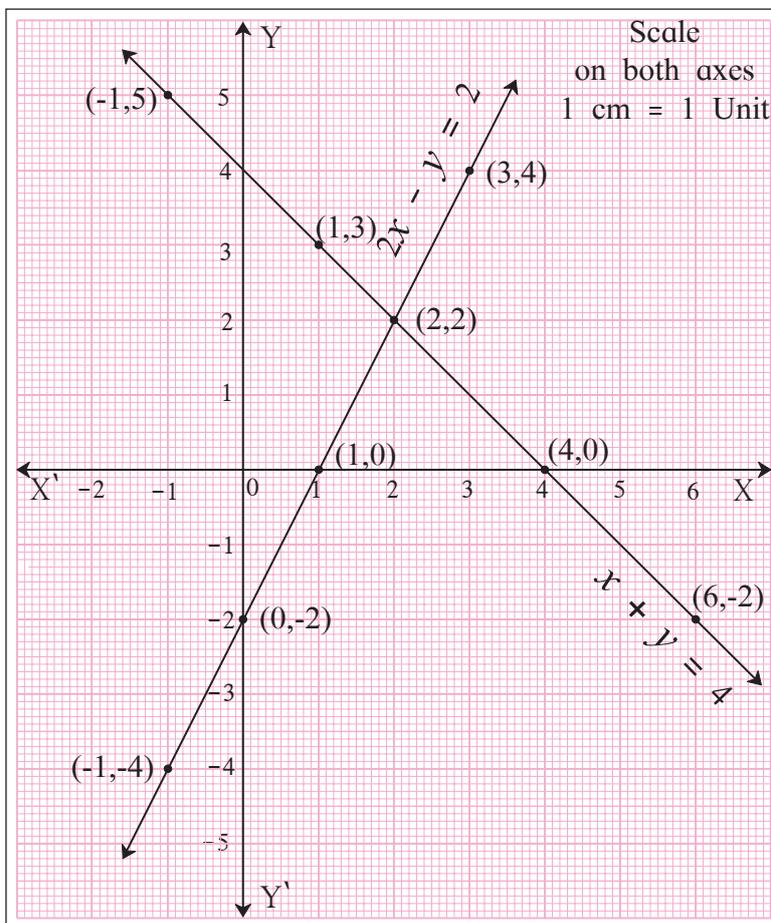
Graphical method

Ex. Let's draw graphs of $x + y = 4$, $2x - y = 2$ and observe them.

$x + y = 4$

x	-1	4	1	6
y	5	0	3	-2
(x,y)	$(-1,5)$	$(4,0)$	$(1,3)$	$(6,-2)$

$2x - y = 2$

x	0	1	3	-1
y	-2	0	4	-4
(x,y)	$(0,-2)$	$(1,0)$	$(3,4)$	$(-1,-4)$

Scale
on both axes
1 cm = 1 Unit

Each point on the graph satisfies the equation. The two lines intersect each other at (2, 2).

Hence ordered pair (2, 2) i.e. $x = 2$, $y = 2$ satisfies the equations $x + y = 4$ and $2x - y = 2$.

The values of variables that satisfy the given equations, give the solution of given equations.

$\therefore$ the solution of given equations $x + y = 4$, $2x - y = 2$ is $x = 2$, $y = 2$.

Let's solve these equations by method of elimination.

$x + y = 4$ . . . (I)

$2x - y = 2$ . . . (II)

Adding equations (I) and (II) we get,

$3x = 6 \therefore x = 2$

substituting this value in equation (I)

$x + y = 4$

$\therefore 2 + y = 4$

$\therefore y = 2$

Activity (I) : Solve the following simultaneous equations by graphical method.

● Complete the following tables to get ordered pairs.

$x - y = 1$

x	0		3	
y		0		-3
(x, y)				

$5x - 3y = 1$

x	2			-4
y		8	-2	
(x, y)				

● Plot the above ordered pairs on the same co-ordinate plane.

● Draw graphs of the equations. ● Note the co-ordinates of the point of intersection of the two graphs. Write solution of these equations.

Activity II : Solve the above equations by method of elimination. Check your solution with the solution obtained by graphical method.

Let's think.

The following table contains the values of x and y co-ordinates for ordered pairs to draw the graph of $5x - 3y = 1$

x	0	$\dfrac{1}{5}$	1	-2
y	$-\dfrac{1}{3}$	0	$\dfrac{4}{3}$	$-\dfrac{11}{3}$
(x, y)	$\left(0, -\dfrac{1}{3}\right)$	$\left(\dfrac{1}{5}, 0\right)$	$\left(1, \dfrac{4}{3}\right)$	$\left(-2, -\dfrac{11}{3}\right)$

● Is it easy to plot these points ? ● Which precaution is to be taken to find ordered pairs so that plotting of points becomes easy ?

Practice Set 1.2

1. Complete the following table to draw graph of the equations –

(I) $x + y = 3$ (II) $x - y = 4$

$x + y = 3$

x	3	☐	☐
y	☐	5	3
(x, y)	(3, 0)	☐	(0, 3)

$x - y = 4$

x	☐	-1	0
y	0	☐	-4
(x, y)	☐	☐	(0, -4)

2. Solve the following simultaneous equations graphically.

(1) $x + y = 6$; $x - y = 4$ (2) $x + y = 5$; $x - y = 3$

(3) $x + y = 0$; $2x - y = 9$ (4) $3x - y = 2$; $2x - y = 3$

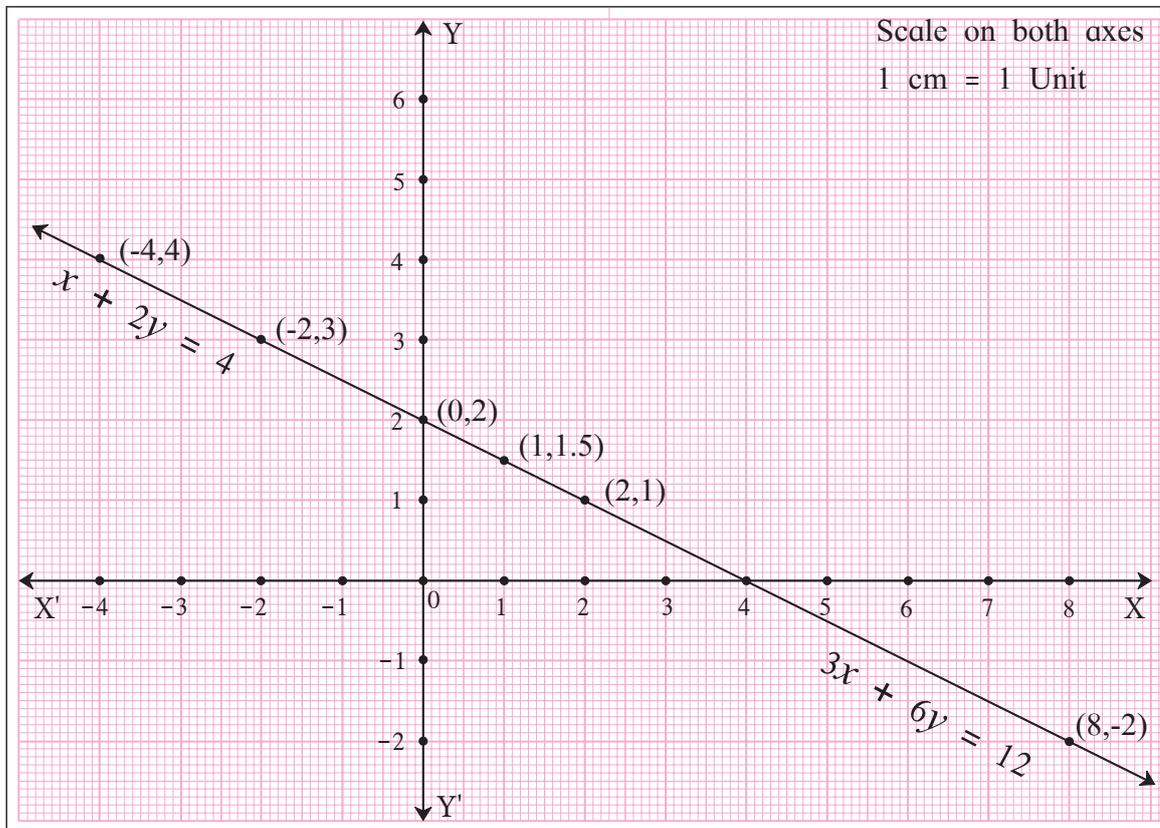(5) $3x - 4y = -7$; $5x - 2y = 0$ (6)★ $2x - 3y = 4$; $3y - x = 4$

To solve simultaneous equations $x + 2y = 4$; $3x + 6y = 12$ graphically, following are the ordered pairs.

$x + 2y = 4$

x	-2	0	2
y	3	2	1
(x, y)	$(-2, 3)$	$(0, 2)$	$(2, 1)$

$3x + 6y = 12$

x	-4	1	8
y	4	1.5	-2
(x, y)	$(-4, 4)$	$(1, 1.5)$	$(8, -2)$

Plotting the above ordered pairs, graph is drawn. Observe it and find answers of the following questions.

(1) Are the graphs of both the equations different or same ?

(2) What are the solutions of the two equations $x + 2y = 4$ and $3x + 6y = 12$? How many solutions are possible ?

(3) What are the relations between coefficients of x, coefficients of y and constant terms in both the equations ?

(4) What conclusion can you draw when two equations are given but the graph is only one line ?

Now let us consider another example.

Draw graphs of $x - 2y = 4$, $2x - 4y = 12$ on the same co-ordinate plane. Observe it. Think of the realation between the coefficients of x, coefficients of y and the constant terms and draw the inference.
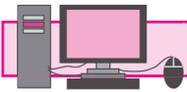
ICT Tools or Links.

Use Geogebra software, draw X- axis, Y-axis. Draw graphs of simultaneous equations.

Let's learn.

Determinant

$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ is a determinant. (a, b), (c, d) are rows and $\begin{pmatrix} a \\ c \end{pmatrix}$, $\begin{pmatrix} b \\ d \end{pmatrix}$ are columns.

Degree of this determinant is 2, because there are 2 elements in each column and 2 elements in each row. Determinant represents a number which is $(ad-bc)$.

i.e. $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad\text{-}bc$

$ad\text{-}bc$ is the value of determinant $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$

Determinants, usually, are represented with capital letters as A, B, C, D, ... etc.

ᘓᘓᘓ **Solved Examples** ᘓᘓᘓ

Ex. Find the values of the following determinants.

(1) A = $\begin{vmatrix} 5 & 3 \\ 7 & 9 \end{vmatrix}$ (2) N = $\begin{vmatrix} -8 & -3 \\ 2 & 4 \end{vmatrix}$ (3) B = $\begin{vmatrix} 2\sqrt{3} & 9 \\ 2 & 3\sqrt{3} \end{vmatrix}$

Solution :

(1) $A = \begin{vmatrix} 5 & 3 \\ 7 & 9 \end{vmatrix} = (5 \times 9) - (3 \times 7) = 45 - 21 = 24$

(2) $N = \begin{vmatrix} -8 & -3 \\ 2 & 4 \end{vmatrix} = [(-8) \times (4)] - [(-3) \times 2)] = -32 - (-6)$

$$= -32 + 6 = -26$$

(3) $B = \begin{vmatrix} 2\sqrt{3} & 9 \\ 2 & 3\sqrt{3} \end{vmatrix} = [2\sqrt{3} \times 3\sqrt{3})] - [2 \times 9)] = 18 - 18 = 0$

Let's learn.

Determinant method (Cramer's Rule)

Using determinants, simultaneous equaions can be solved easily and in less space. This method is known as determinant method. This method was first given by a Swiss mathematician Gabriel Cramer, so it is also known as Cramer's method.

To use Cramer's method, the equations are written as $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$.

$$a_1x + b_1 y = c_1 \quad \dots \quad \text{(I)}$$

$$a_2x + b_2 y = c_2 \quad \dots \quad \text{(II)}$$

Here x and y are variables, a_1, b_1, c_1 and a_2, b_2, c_2 are real numbers, $a_1b_2 - a_2b_1 \neq 0$

Now let us solve these equations.

Multiplying equation (I) by b_2.

$$a_1 b_2 x + b_1 b_2 y = c_1 b_2 \quad \dots \quad \text{(III)}$$

Multiplying equation (II) by b_1.

$$a_2 b_1 x + b_2 b_1 y = c_2 b_1 \quad \dots \quad \text{(IV)}$$

Subtracting equation (IV) from (III)

$$a_1 b_2 x + b_1 b_2 y = c_1 b_2$$

$$\underline{\ \underset{-}{a_2 b_1 x}\ \underset{-}{\ b_2 b_1 y}\ =\ \underset{-}{c_2 b_1}}$$

$$(a_1 b_2 - a_2 b_1)\, x = c_1 b_2 - c_2 b_1$$

$$x = \frac{c_1 b_2 - c_2 b_1}{a_1 b_2 - a_2 b_1} \quad \ldots \quad (V)$$

Similarly $\quad y = \dfrac{a_1 c_2 - a_2 c_1}{a_1 b_2 - a_2 b_1} \quad \ldots \quad (VI)$

To remember and write the expressions

$c_1 b_2 - c_2 b_1,\ \ a_1 b_2 - a_2 b_1,\ \ a_1 c_2 - a_2 c_1$ we use the determinants.

Now $a_1 x + b_1 y = c_1$ $\qquad$ We can write 3 columns. $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix},\ \begin{pmatrix} b_1 \\ b_2 \end{pmatrix},\ \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$

and $\quad a_2 x + b_2 y = c_2$

The values x, y in equation (V), (VI) are written using determinants as follows

$$x = \frac{c_1 b_2 - c_2 b_1}{a_1 b_2 - a_2 b_1} = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}},$$

$$y = \frac{a_1 c_2 - a_2 c_1}{a_1 b_2 - a_2 b_1} = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

To remember let us denote $D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix},\quad D_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix},\quad D_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$

$$\therefore\ x = \frac{D_x}{D},\quad y = \frac{D_y}{D}$$

For writting D, Dx, Dy remember the order of columns $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix},\ \begin{pmatrix} b_1 \\ b_2 \end{pmatrix},\ \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}.$

From the equations,

$$a_1 x + b_1 y = c_1$$

and $a_2 x + b_2 y = c_2$ we get the columns $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$, $\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$, $\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$.

- In D the column of constants $\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$ is omitted.

- In D_x the column of the coefficients of x, $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ is replaced by $\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$.

- In D_y the column of the coefficients of y, $\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$ is replaced by $\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$.

Let's remember!

Cramer's method to solve simultaneous equations.

Write given equations in the form $ax + by = c$.

Find the values of determinants D, D_x and D_y

Using, $x = \dfrac{D_x}{D}$ and $y = \dfrac{D_y}{D}$

find values of x, y.

Gabriel Cramer

(31 July, 1704 to 4 January, 1752)

This Swiss mathematician was born in Geneva. He was very well versed in mathematics, since childhood. At the age of eighteen, he got a doctorate. He was a professor in Geneva.

Ex. (1) Solve the following simultaneous equations using Cramer's Rule.

$$5x + 3y = -11 \; ; \; 2x + 4y = -10$$

Solution : Given equations

$$5x + 3y = -11$$

$$2x + 4y = -10$$

$$D = \begin{vmatrix} 5 & 3 \\ 2 & 4 \end{vmatrix} \quad = (5 \times 4) - (2 \times 3) = 20 - 6 = 14$$

$$D_x = \begin{vmatrix} -11 & 3 \\ -10 & 4 \end{vmatrix} = (-11) \times 4 - (-10) \times 3 = -44 - (-30)$$

$$= -44 + 30 = -14$$

$$D_y = \begin{vmatrix} 5 & -11 \\ 2 & -10 \end{vmatrix} = 5 \times (-10) - 2 \times (-11) = -50 - (-22)$$

$$= -50 + 22 = -28$$

$$x = \frac{D_x}{D} = \frac{-14}{14} = -1 \qquad \text{and } y = \frac{D_y}{D} = \frac{-28}{14} = -2$$

$$\therefore (x, y) = (-1, -2) \text{ is the solution.}$$

Activity 1 : To solve the simultaneous equations by determinant method, fill in the blanks

$$y + 2x - 19 = 0 \; ; \; 2x - 3y + 3 = 0$$

Solution : Write the given equations in the form $ax + by = c$

$$2x + y = 19$$

$$2x - 3y = -3$$

$$D = \begin{vmatrix} \boxed{} & \boxed{} \\ 2 & -3 \end{vmatrix} = [\boxed{} \times (-3)] - [2 \times (\boxed{})] = \boxed{} - (\boxed{})$$

$$= \boxed{} - \boxed{} = \boxed{}$$

$$D_x = \begin{vmatrix} 19 & \boxed{} \\ \boxed{} & -3 \end{vmatrix} = [19 \times (\boxed{})] - [(\boxed{}) \times (\boxed{})] = \boxed{} - \boxed{}$$

$$= \boxed{}$$

$$D_y = \begin{vmatrix} \boxed{} & 19 \\ 2 & \boxed{} \end{vmatrix} = [(\boxed{}) \times (\boxed{})] - [(\boxed{}) \times (\boxed{})]$$

$$= \boxed{} - \boxed{} = \boxed{}$$

By Cramer's Rule –

$$x = \frac{D_x}{D}$$

$$\therefore x = \frac{\boxed{}}{\boxed{}} = \boxed{}$$

$$y = \frac{D_y}{D}$$

$$y = \frac{\boxed{}}{\boxed{}} = \boxed{}$$

$\therefore (x, y) = (\boxed{}, \boxed{})$ is the solution of the given equations.

Activity 2 : Complete the following activity –

$3x-2y=3$ $\qquad$ $2x+y=16$

Find the values of determinants in the given equations

$$D = \begin{vmatrix} \end{vmatrix} = \boxed{} \qquad D_x = \begin{vmatrix} \end{vmatrix} = \boxed{} \qquad D_y = \begin{vmatrix} \end{vmatrix} = \boxed{}$$

Values according to Cramer's Rule

$$x = \frac{\boxed{}}{\boxed{}} = \boxed{} \qquad\qquad y = \frac{\boxed{}}{\boxed{}} = \boxed{}$$
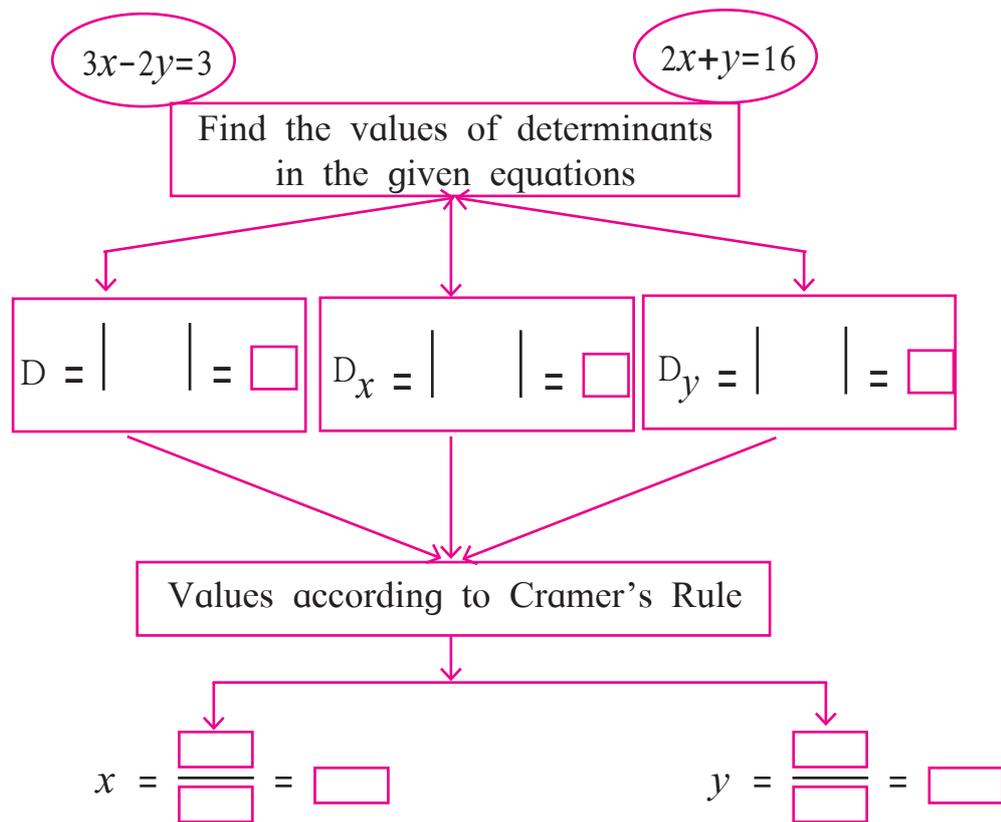
$\therefore (x, y) = (\boxed{}, \boxed{})$ is the solution.

- What is the nature of solution if D = 0 ?
- What can you say about lines if common solution is not possible?

Practice Set 1.3

1. Fill in the blanks with correct number

$$\begin{vmatrix} 3 & 2 \\ 4 & 5 \end{vmatrix} = 3 \times \boxed{} - \boxed{} \times 4 \quad = \boxed{} - 8 \quad = \boxed{}$$

2. Find the values of following determinants.

(1) $\begin{vmatrix} -1 & 7 \\ 2 & 4 \end{vmatrix}$
(2) $\begin{vmatrix} 5 & 3 \\ -7 & 0 \end{vmatrix}$
(3) $\begin{vmatrix} \dfrac{7}{3} & \dfrac{5}{3} \\ \dfrac{3}{2} & \dfrac{1}{2} \end{vmatrix}$

3. Solve the following simultaneous equations using Cramer's rule.

(1) $3x - 4y = 10$; $4x + 3y = 5$ (2) $4x + 3y - 4 = 0$; $6x = 8 - 5y$

(3) $x + 2y = -1$; $2x - 3y = 12$ (4) $6x - 4y = -12$; $8x - 3y = -2$

(5) $4m + 6n = 54$; $3m + 2n = 28$ (6) $2x + 3y = 2$; $x - \dfrac{y}{2} = \dfrac{1}{2}$

> **Let's learn.**

(Equations reducible to a pair of linear equations in two variables)

Activity : Complete the following table.

Equation	No. of variables	whether linear or not
$\dfrac{3}{x} - \dfrac{4}{y} = 8$	2	Not linear
$\dfrac{6}{x-1} + \dfrac{3}{y-2} = 0$	$\boxed{}$	$\boxed{}$
$\dfrac{7}{2x+1} + \dfrac{13}{y+2} = 0$	$\boxed{}$	$\boxed{}$
$\dfrac{14}{x+y} + \dfrac{3}{x-y} = 5$	$\boxed{}$	$\boxed{}$

Let's think.

In the above table the equations are not linear. Can you convert the equations into linear equations ?

Let's remember!

We can create new variables making a proper change in the given variables. Substituting the new variables in the given non-linear equations, we can convert them in linear equations.

Also remember that the denominator of any fraction of the form $\frac{m}{n}$ cannot be zero.

$\mathscr{SSS}$ Solved examples $\mathscr{SSS}$

Solve:

Ex. (1) $\frac{4}{x} + \frac{5}{y} = 7$; $\frac{3}{x} + \frac{4}{y} = 5$

Solution : $\frac{4}{x} + \frac{5}{y} = 7$; $\frac{3}{x} + \frac{4}{y} = 5$

$$4\left(\frac{1}{x}\right) + 5\left(\frac{1}{y}\right) = 7 \ldots \text{(I)}$$

$$3\left(\frac{1}{x}\right) + 4\left(\frac{1}{y}\right) = 5 \ldots \text{(II)}$$

Replacing $\left(\frac{1}{x}\right)$ by m and $\left(\frac{1}{y}\right)$ by n in equations (I) and (II), we get

$4m + 5n = 7 \ldots$ (III)

$3m + 4n = 5 \ldots$ (IV)

On solving these equations we get

$m = 3$, $n = -1$

Now, $m = \frac{1}{x}$ $\therefore 3 = \frac{1}{x}$ $\therefore x = \frac{1}{3}$

$n = \frac{1}{y}$ $\therefore -1 = \frac{1}{y}$ $\therefore y = -1$

$\therefore$ Solution of given simultaneous equations is $(x, y) = (\frac{1}{3}, -1)$

Ex.(2) $\dfrac{4}{x-y} + \dfrac{1}{x+y} = 3$; $\dfrac{2}{x-y} - \dfrac{3}{x+y} = 5$

Solution : $\dfrac{4}{x-y} + \dfrac{1}{x+y} = 3$; $\dfrac{2}{x-y} - \dfrac{3}{x+y} = 5$

$$4\left(\dfrac{1}{x-y}\right) + 1\left(\dfrac{1}{x+y}\right) = 3 \ . \ . \ . \quad \text{(I)}$$

$$2\left(\dfrac{1}{x-y}\right) - 3\left(\dfrac{1}{x+y}\right) = 5 \ . \ . \ . \quad \text{(II)}$$

Replacing $\left(\dfrac{1}{x-y}\right)$ by a and $\left(\dfrac{1}{x+y}\right)$ by b we get

$4a + b = 3 \ . \ . \ .$ (III)

$2a - 3b = 5 \ . \ . \ .$ (IV)

On solving these equations we get, $a = 1 \ \ b = -1$

But $a = \left(\dfrac{1}{x-y}\right)$, $b = \left(\dfrac{1}{x+y}\right)$

$\therefore \left(\dfrac{1}{x-y}\right) = 1, \left(\dfrac{1}{x+y}\right) = -1$

$\therefore \ \ x - y = 1 \ . \ . \ .$ (V)

$\ \ \ \ \ x + y = -1 \ . \ . \ .$ (VI)

Solving equation (V) and (VI) we get $x = 0, \ y = -1$

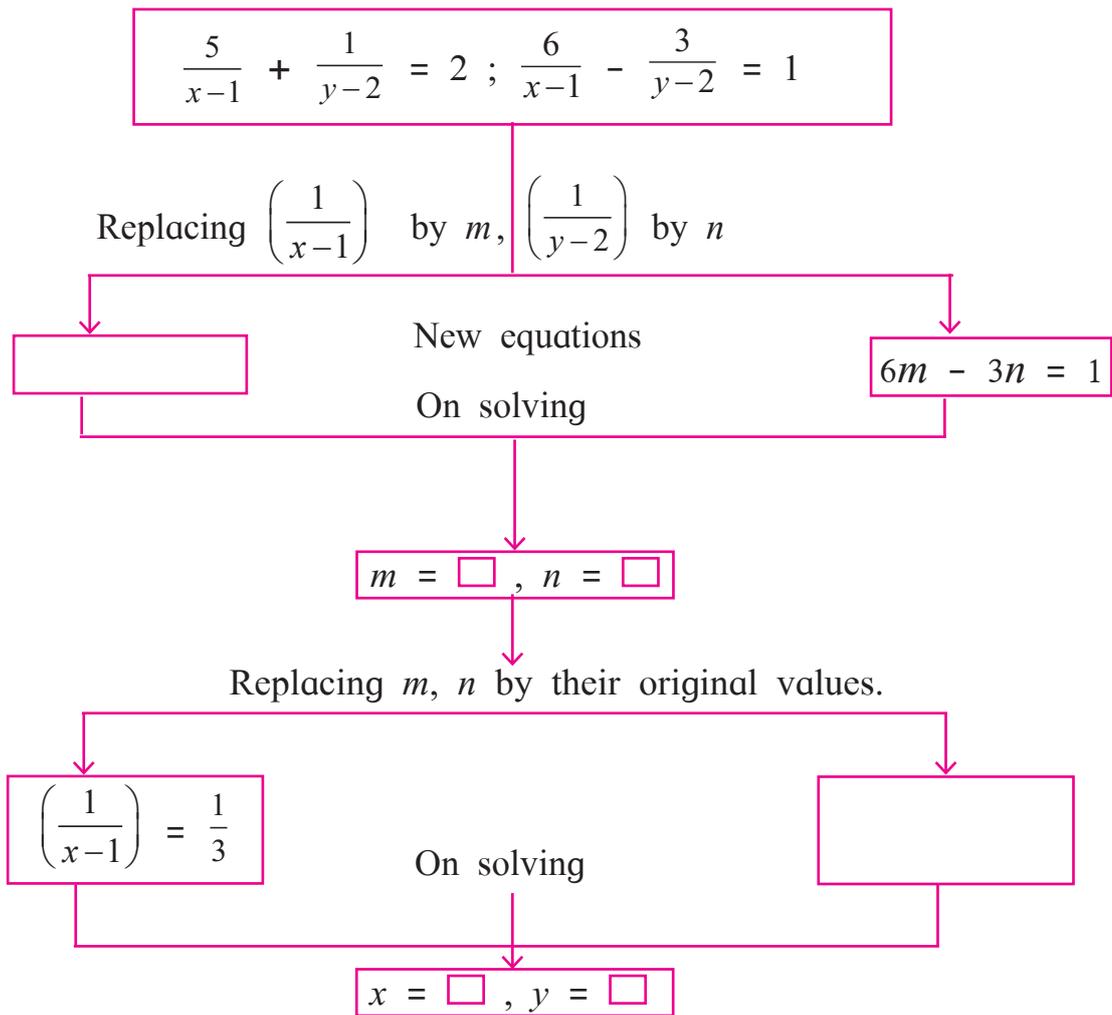$\therefore$ Solution of the given equations is $(x, \ y) = (0, \ -1)$

Let's think.

In the above examples the simultaneous equations obtained by transformation are solved by elimination method.

If you solve these equations by graphical method and by Cramer's rule will you get the same answers ? Solve and check it.

Activity : To solve given equations fill the boxes below suitably.

$$\frac{5}{x-1} + \frac{1}{y-2} = 2 \; ; \; \frac{6}{x-1} - \frac{3}{y-2} = 1$$

Replacing $\left(\dfrac{1}{x-1}\right)$ by m, $\left(\dfrac{1}{y-2}\right)$ by n

New equations

	$6m - 3n = 1$

On solving

$$m = \square \;,\; n = \square$$

Replacing m, n by their original values.

$$\left(\frac{1}{x-1}\right) = \frac{1}{3}$$

On solving

$$x = \square \;,\; y = \square$$

$\therefore$ $(x, y) = (\quad , \quad)$ is the solution of the given simultaneous equations.

Practice Set 1.4

1. Solve the following simultaneous equations.

(1) $\dfrac{2}{x} - \dfrac{3}{y} = 15 \; ; \; \dfrac{8}{x} + \dfrac{5}{y} = 77$

(2) $\dfrac{10}{x+y} + \dfrac{2}{x-y} = 4 \; ; \; \dfrac{15}{x+y} - \dfrac{5}{x-y} = -2$

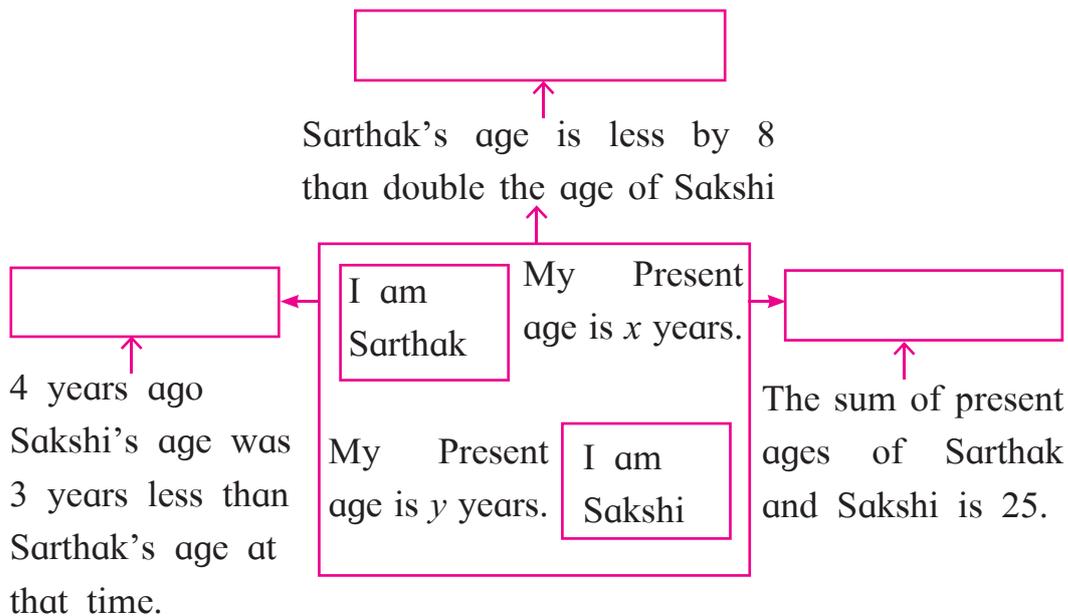(3) $\dfrac{27}{x-2} + \dfrac{31}{y+3} = 85 \; ; \; \dfrac{31}{x-2} + \dfrac{27}{y+3} = 89$

(4) $\dfrac{1}{3x+y} + \dfrac{1}{3x-y} = \dfrac{3}{4} \; ; \; \dfrac{1}{2(3x+y)} - \dfrac{1}{2(3x-y)} = -\dfrac{1}{8}$

Application of Simultaneous equations

Activity : There are some instructions given below. Frame the equations from the information and write them in the blank boxes shown by arrows.

```
┌──────────────────────┐
│                      │
└──────────────────────┘
           ↑
Sarthak's age is less by 8
than double the age of Sakshi
                    ↑
┌─────────────┐  ┌─────────────────────────┐  ┌──────────────┐
│             │←─│ I am      My    Present │─→│              │
└─────────────┘  │ Sarthak   age is x years.│  └──────────────┘
      ↑          │                          │        ↑
4 years  ago     │ My     Present ┌───────┐ │  The sum of present
Sakshi's age was │ age is y years.│ I am  │ │  ages   of   Sarthak
3 years less than│                │ Sakshi│ │  and Sakshi is 25.
Sarthak's age at └────────────────┴───────┴─┘
that time.
```

Ex. (1) The perimeter of a rectangle is 40 cm. The length of the rectangle is more than double its breadth by 2. Find length and breadth.

Solution : Let length of rectangle be x cm and breadth be y cm.

From first condition –

$$2(x + y) = 40$$

$$x + y = 20 \ldots \quad (I)$$

From 2nd condition –

$$x = 2y + 2$$

$$\therefore x - 2y = 2 \ldots \quad (II)$$

Let's solve eq. (I), (II) by determinant method

$$x + y = 20$$

$$x - 2y = 2$$

$$D = \begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} = [1 \times (-2)] - (1 \times 1) = -2 - 1 = -3$$

$$D_x = \begin{vmatrix} 20 & 1 \\ 2 & -2 \end{vmatrix} = [20 \times (-2)] - (1 \times 2) = -40 - 2 = -42$$

$$D_y = \begin{vmatrix} 1 & 20 \\ 1 & 2 \end{vmatrix} = (1 \times 2) - (20 \times 1) = 2 - 20 = -18$$

$$x = \frac{D_x}{D} \text{ and } y = \frac{D_y}{D}$$

$$\therefore x = \frac{-42}{-3} \text{ and } y = \frac{-18}{-3}$$

$$\therefore x = 14, \quad y = 6$$

$\therefore$ Length of the rectangle is 14 cm and breadth is 6 cm.

Ex. (2)

Sale ! Sale !! Sale !!! only for 2 days

I have some analogue wrist watches and some digital wrist watches. I am going to sell them at a discount

Sale of 1st day

Analogue watch = 11

Digital watch = 6

Received amount = ₹ 4330

Sale of the 2nd day

Analogue watch = 22

Digital watch = 5

Received amount = ₹ 7330

Find selling price of wrist watch of each type.

Solution : Let selling price of each analogue watch be ₹ x

Selling price of each digital watch be ₹ y

From first condition –

$$11x + 6y = 4330 \ldots \quad \text{(I)}$$

from 2nd condition –

$$22x + 5y = 7330 \ldots \quad \text{(II)}$$

multiplying equation (I) by 2 we get,

$$22x + 12y = 8660 \ldots \quad \text{(III)}$$

subtract equation (III) from equation (II).

$$22x + 5y = 7330 \ldots \quad \text{(II)}$$
$$- \quad 22x + 12y = 8660 \ldots \quad \text{(III)}$$
$$\overline{\qquad\qquad\qquad\qquad}$$
$$-7y = -1330$$

$$\therefore \quad y = 190$$

Substitute this value of y in equation (I)

$$11x + 6y = 4330$$

$$\therefore 11x + 6(190) = 4330$$

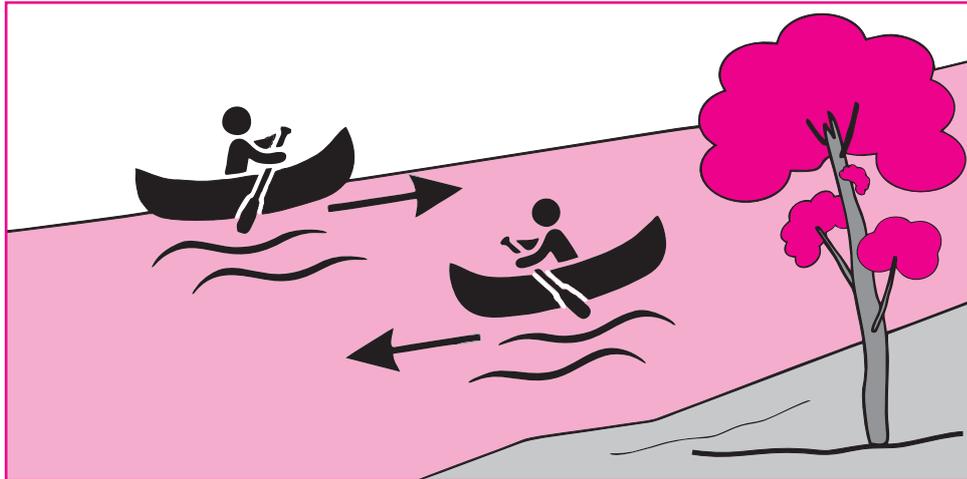$$\therefore \quad 11x + 1140 = 4330$$

$$\therefore \qquad 11x = 3190$$

$$\therefore \qquad x = 290$$

$\therefore$ selling price of each analogue watch is ₹ 290 and

of each digital watch is ₹ 190.

Ex. (3)

| A boat travels 16 km upstream and 24 km downstream in 6 hours. | The same boat travels 36 km upstream and 48 km downstream in 13 hours. |

Find the speed of water current and speed of boat in still water.

Solution : Let the speed of the boat in still water be x km/hr and the speed of water current be y km/hr

$\therefore$ speed of boat in downstream = $(x + y)$ km/hr.

and that in upstream = $(x - y)$ km/hr.

Now distance = speed × time $\therefore$ time = $\dfrac{\text{distance}}{\text{speed}}$

Time taken by the boat to travel 16 km upstream = $\dfrac{16}{x-y}$ hours and it takes $\dfrac{24}{x+y}$ hours to travel 24 km downstream.

from first condition –

$$\dfrac{16}{x-y} + \dfrac{24}{x+y} = 6 \ . \ . \ . \ (I)$$

from 2nd condition

$$\dfrac{36}{x-y} + \dfrac{48}{x+y} = 13 \ . \ . \ . \ (II)$$

By replacing $\dfrac{1}{x-y}$ by m and $\dfrac{1}{x+y}$ by n we get

$16m + 24n = 6 \ . \ . \ . \ (III)$

$36m + 48n = 13 \ . \ . \ . \ (IV)$

Solving equations (III) and (IV) $m = \dfrac{1}{4}$, $n = \dfrac{1}{12}$

Repalcing m, n by their original values we get

$x - y = 4$ . . . (V) $x + y = 12$ . . . (VI)

Solving equations (V), (VI) we get $x = 8$, $y = 4$

∴ speed of the boat in still water is 8 km/hr. and speed of water current is 4 km/hr.

Ex. (4) A certain amount is equally distributed among certain number of students. Each would get ₹ 2 less if 10 students were more and each would get ₹ 6 more if 15 students were less. Find the number of students and the amount distributed.

Solution : Let the number of students be x and amount given to each student be ₹ y.

∴ Total amount distributed is xy

From the first condition we get,

$(x + 10)(y - 2) = xy$

∴ $xy - 2x + 10y - 20 = xy$

∴ $- 2x + 10y = 20$

∴ $- x + 5y = 10$ . . . (I)

From the 2nd condition we get,

$(x - 15)(y + 6) = xy$

∴ $xy + 6x - 15y - 90 = xy$

∴ $6x - 15y = 90$

∴ $2x - 5y = 30$ . . . (II)

Adding equations (I) and (II)

$$
\begin{aligned}
&- x + 5y = 10 \\
+\quad &2x - 5y = 30 \\
\hline
&x = 40
\end{aligned}
$$

Substitute this value of x in equation (I)

$- x + 5y = 10$

∴ $- 40 + 5y = 10$

∴ $5y = 50$

$$\therefore \quad y = 10$$

Total amount distributed is $= xy = 40 \times 10 = ₹\ 400$.

$\therefore \quad ₹\ 400$ distributed equally among 40 students.

Ex. (5) A three digit number is equal to 17 times the sum of its digits; If the digits are reversed, the new number is 198 more than the old number ; also the sum of extreme digits is less than the middle digit by unity. Find the original number.

Solution : Let the digit in hundreds place be x and that in unit place be y.

H	T	unit
x	$x + y + 1$	y

$\therefore$ the three digit number is $100x + 10(x + y + 1) + y$

$= 100x + 10x + 10y + 10 + y = 110x + 11y + 10$

the sum of the digits in the given number $= x + (x + y + 1) + y = 2x + 2y + 1$

$\therefore$ From first condition

Given number $= 17 \times$ (sum of the digits)

$\therefore \ 110x + 11y + 10 = 17 \times (2x + 2y + 1)$

$\therefore \ 110x + 11y + 10 = 34x + 34y + 17$

$\therefore \ 76x - 23y = 7 \ . . . $ (I)

The number obtained by reversing the digits

$= 100y + 10(x + y + 1) + x = 110y + 11x + 10$

Given number $= 110x + 11y + 10$

From 2nd condition, Given number $+ 198 = $ new number.

$110x + 11y + 10 + 198 = 110y + 11x + 10$

$99x - 99y = -198$

$x - y = -2$

$\therefore \ x = y - 2 \ . . . $ (II)

Substitute this value of x in equation (I).

$\therefore \ 76(y - 2) - 23y = 7$

$\therefore \ 76y - 152 - 23y = 7$

$53y = 159$

$\therefore$ y = 3 $\therefore$ the digit in units place is = 3

Substitute this value in equation (II)

$x = y - 2$

$\therefore$ $x = 3 - 2$ = 1

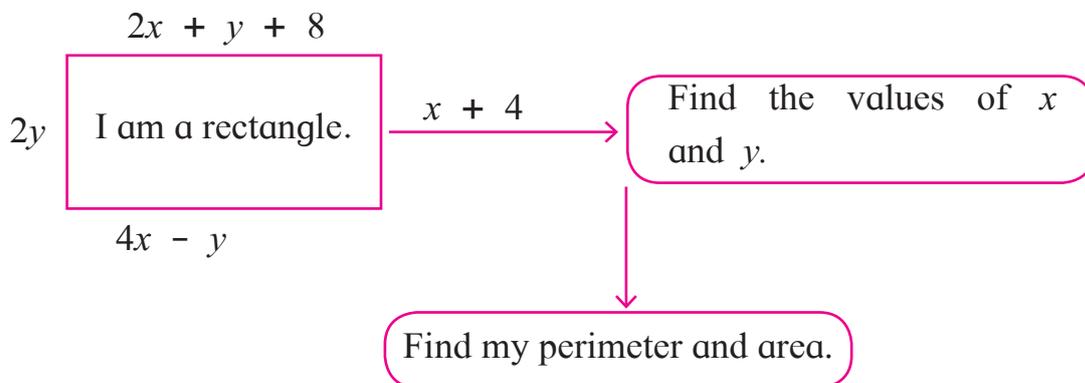$\therefore$ $x = 1$ $\therefore$ The digit in hundred's place is 1

the digit in ten's place is 3 + 1 + 1 = 5

$\therefore$ the number is 153.

Practice Set 1.5

(1) Two numbers differ by 3. The sum of twice the smaller number and thrice the greater number is 19. Find the numbers.

(2) Complete the following.

$$2x + y + 8$$

$2y$	I am a rectangle.

$x + 4$ → Find the values of x and y.

$4x - y$

↓

Find my perimeter and area.

(3) The sum of father's age and twice the age of his son is 70. If we double the age of the father and add it to the age of his son the sum is 95. Find their present ages.

(4) The denominator of a fraction is 4 more than twice its numerator. Denominator becomes 12 times the numerator, if both the numerator and the denominator are reduced by 6. Find the fraction.

(5) Two types of boxes A, B are to be placed in a truck having capacity of 10 tons. When 150 boxes of type A and 100 boxes of type B are loaded in the truck, it weighes 10 tons. But when 260 boxes of type A are loaded in the truck, it can still accommodate 40 boxes of type B, so that it is fully loaded. Find the weight of each type of box.

(6)★ Out of 1900 km, Vishal travelled some distance by bus and some by aeroplane. Bus travels with average speed 60 km/hr and the average speed of aeroplane is 700 km/hr. It takes 5 hours to complete the journey. Find the distance, Vishal travelled by bus.

Problem Set - 1

1. Choose correct alternative for each of the following questions

(1) To draw graph of $4x + 5y = 19$, Find y when $x = 1$.

    (A) 4   (B) 3   (C) 2   (D) -3

(2) For simultaneous equations in variables x and y, $D_x = 49$, $D_y = -63$, $D = 7$ then what is x ?

    (A) 7   (B) -7   (C) $\dfrac{1}{7}$ (D) $\dfrac{-1}{7}$

(3) Find the value of $\begin{vmatrix} 5 & 3 \\ -7 & -4 \end{vmatrix}$

    (A) -1   (B) -41   (C) 41   (D) 1

(4) To solve $x + y = 3$; $3x - 2y - 4 = 0$ by determinant method find D.

    (A) 5   (B) 1   (C) -5   (D) -1

(5) $ax + by = c$ and $mx + ny = d$ and $an \neq bm$ then these simultaneous equations have -

    (A) Only one common solution.   (B) No solution.

    (C) Infinite number of solutions.   (D) Only two solutions.

2. Complete the following table to draw the graph of $2x - 6y = 3$

x	-5	☐
y	☐	0
(x, y)	☐	☐

3. Solve the following simultaneous equations graphically.

(1) $2x + 3y = 12$; $x - y = 1$

(2) $x - 3y = 1$; $3x - 2y + 4 = 0$

(3) $5x - 6y + 30 = 0$; $5x + 4y - 20 = 0$

(4) $3x - y - 2 = 0$; $2x + y = 8$

(5) $3x + y = 10$; $x - y = 2$

4. Find the values of each of the following determinants.

(1) $\begin{vmatrix} 4 & 3 \\ 2 & 7 \end{vmatrix}$ (2) $\begin{vmatrix} 5 & -2 \\ -3 & 1 \end{vmatrix}$ (3) $\begin{vmatrix} 3 & -1 \\ 1 & 4 \end{vmatrix}$

5. Solve the following equations by Cramer's method.

(1) $6x - 3y = -10$; $3x + 5y - 8 = 0$

(2) $4m - 2n = -4$; $4m + 3n = 16$

(3) $3x - 2y = \dfrac{5}{2}$; $\dfrac{1}{3}x + 3y = -\dfrac{4}{3}$

(4) $7x + 3y = 15$; $12y - 5x = 39$

(5) $\dfrac{x+y-8}{2} = \dfrac{x+2y-14}{3} = \dfrac{3x-y}{4}$

6. Solve the following simultaneous equations.

(1) $\dfrac{2}{x} + \dfrac{2}{3y} = \dfrac{1}{6}$; $\dfrac{3}{x} + \dfrac{2}{y} = 0$ (2) $\dfrac{7}{2x+1} + \dfrac{13}{y+2} = 27$; $\dfrac{13}{2x+1} + \dfrac{7}{y+2} = 33$

(3) $\dfrac{148}{x} + \dfrac{231}{y} = \dfrac{527}{xy}$; $\dfrac{231}{x} + \dfrac{148}{y} = \dfrac{610}{xy}$ (4) $\dfrac{7x-2y}{xy} = 5$; $\dfrac{8x+7y}{xy} = 15$

(5) $\dfrac{1}{2(3x+4y)} + \dfrac{1}{5(2x-3y)} = \dfrac{1}{4}$; $\dfrac{5}{(3x+4y)} - \dfrac{2}{(2x-3y)} = -\dfrac{3}{2}$

7. Solve the following word problems.

(1) A two digit number and the number with digits interchanged add up to 143. In the given number the digit in unit's place is 3 more than the digit in the ten's place. Find the original number.

Let the digit in unit's place is x

and that in the ten's place is y

∴ the number = $\boxed{}\, y + x$

The number obtained by interchanging the digits is $\boxed{}\, x + y$

According to first condition two digit number + the number obtained by interchanging the digits = 143

∴ $\boxed{10y + x}$ + $\boxed{}$ = 143

∴ $\boxed{}\, x + \boxed{}\, y = 143$

$x + y = \boxed{}$ . . . . . (I)

From the second condition,

digit in unit's place = digit in the ten's place + 3

∴ $x = \boxed{} + 3$

∴ $x - y = 3$ . . . . . (II)

Adding equations (I) and (II)

$$2x = \boxed{}$$

$$\boxed{x = 8}$$

Putting this value of x in equation (I)

$$x + y = 13$$

$$8 + \boxed{} = 13$$

$$\therefore \quad y = \boxed{}$$

The original number is $10 \, y + x$

$$= \boxed{} + 8$$

$$= 58$$

(2) Kantabai bought $1\frac{1}{2}$ kg tea and 5 kg sugar from a shop. She paid ₹ 50 as return fare for rickshaw. Total expense was ₹ 700. Then she realised that by ordering online the goods can be bought with free home delivery at the same price. So next month she placed the order online for 2 kg tea and 7 kg sugar. She paid ₹ 880 for that. Find the rate of sugar and tea per kg.

(3) To find number of notes that Anushka had, complete the following activity.

Suppose that Anushka had $\boxed{x}$ notes of ₹ 100 and $\boxed{y}$ notes of ₹ 50 each

Anushka got ₹ 2500/− from Anand as denominations mentioned above ∴ .... equation I	If Anand would have given her the amount by interchanging number of notes, Anushka would have received ₹ 500 less than the previous amount ∴ .... equation II

$$\therefore \text{ The No. of notes } (\boxed{}, \boxed{})$$

(4) Sum of the present ages of Manish and Savita is 31. Manish's age 3 years ago was 4 times the age of Savita. Find their present ages.

(5) ★ In a factory the ratio of salary of skilled and unskilled workers is 5 : 3. Total salary of one day of both of them is ₹ 720. Find daily wages of skilled and unskilled workers.

(6) ★ Places A and B are 30 km apart and they are on a straight road. Hamid travels from A to B on bike. At the same time Joseph starts from B on bike, travels towards A. They meet each other after 20 minutes. If Joseph would have started from B at the same time but in the opposite direction (instead of towards A) Hamid would have caught him after 3 hours. Find the speed of Hamid and Joseph.

2 Quadratic Equations

Let's study.

- Quadratic equation : Introduction
- Methods of solving quadratic equation
- Nature of roots of quadratic equation
- Relation between roots and coefficients
- Applications of quadratic equations

Let's recall.

You have studied polynomials last year. You know types of polynomials according to their degree. When the degree of polynomial is 1 it is called a linear polynomial and if degree of a polynomial is 2 it is called a quadratic polynomial.

Activity : Classify the following polynomials as linear and quadratic.

$$5x + 9, \qquad x^2 + 3x - 5, \quad 3x - 7, \qquad 3x^2 - 5x, \qquad 5x^2$$

$\boxed{\text{Linear polynomials}}$ $\qquad\qquad$ $\boxed{\text{Quadratic polynomials}}$

Now equate the quadratic polynomial to 0 and study the equation we get. Such type of equation is known as quadratic equation. In practical life we may use quadratic equations many times.

Ex. Sanket purchased a rectangular plot having area 200 m². Length of the plot was 10 m more than its breadth. Find the length and the breadth of the plot.

Let the breadth of the plot be x metre.

$\therefore$ Length = $(x + 10)$ metre

Area of rectangle = length × breadth

$\therefore$ 200 = $(x + 10) \times x$

$\therefore$ 200 = $x^2 + 10x$

That is $x^2 + 10x = 200$

$\therefore$ $x^2 + 10x - 200 = 0$

Now, solving equation $x^2 + 10x - 200 = 0$, we will decide the dimensions of the plot.

Let us study how to solve the quadratic equation.

Let's recall.

Activity : $x^2 + 3x - 5$, $3x^2 - 5x$, $5x^2$; Write the polynomials in the index form.

Observe the coefficients and fill in the boxes.

$x^2 + 3x - 5$, $3x^2 - 5x + 0$, $5x^2 + 0x + 0$

✦ Coefficients of x^2 are $\boxed{1}$, $\boxed{3}$ and $\boxed{5}$ these coefficients are non zero.

✦ Coefficients of x are   3, $\boxed{}$ and $\boxed{}$ respectively.

✦ Constants terms are $\boxed{}$, $\boxed{}$ and $\boxed{}$ respectively.

Here constant term of second and third polynomial is zero.

Let's learn.

Standard form of quadratic equation

The equation involving one variable and having 2 as the maximum index of the variable is called the quadratic equation.

General form is $ax^2 + bx + c = 0$

In $ax^2 + bx + c = 0$, a, b, c are real numbers and $a \neq 0$.

$ax^2 + bx + c = 0$ is the general form of quadratic equation.

Activity : Complete the following table

Quadratic Equation	General form	a	b	c
$x^2 - 4 = 0$	$x^2 + 0x - 4 = 0$	1	0	-4
$y^2 = 2y - 7$	. . . .	. . .	. . .	. . .
$x^2 + 2x = 0$	. . . .	. . .	. . .	. . .

🙠🙠🙠 **Solved Examples** 🙠🙠🙠

Ex. (1) Decide which of the following are quadratic equations ?

(1) $3x^2 - 5x + 3 = 0$ (2) $9y^2 + 5 = 0$ (3) $m^3 - 5m^2 + 4 = 0$ (4) $(l + 2)(l - 5) = 0$

Solution : (1) In the equation $3x^2 - 5x + 3 = 0$, x is the only variable and maximum index of the variable is 2

∴ It is a quadratic equation.

(2) In the equation $9y^2 + 5 = 0$, ☐ is the only variable and maximum index of the variable is ☐

∴ It ☐ a quadratic equation.

(3) In the equation $m^3 - 5m^2 + 4 = 0$, ☐ is the only variable but maximum index of the variable is not 2.

∴ It ☐ a quadratic equation.

(4) $(l + 2)(l - 5) = 0$

∴ $l(l - 5) + 2(l - 5) = 0$

∴ $l^2 - 5l + 2l - 10 = 0$

∴ $l^2 - 3l - 10 = 0$, In this equation ☐ is the only variable and maximum index of the variable is ☐ .

∴ It ☐ a quadratic equation.

Let's learn.

Roots of a quadratic equation

In the previous class you have studied that if value of the polynomial is zero for $x = a$ then $(x - a)$ is a factor of that polynomial. That is if p(x) is a polynomial and p(a) = 0 then $(x - a)$ is a factor of p(x). In this case 'a' is the root or solution of p(x) = 0

For Example ,

Let $x = -6$ in the polynomial $x^2 + 5x - 6$

$x^2 + 5x - 6 = (-6)^2 + 5 \times (-6) - 6$

$= 36 - 30 - 6 \quad = 0$

∴ $x = -6$ is a solution of the equation.

Hence -6 is one root of the equation

$x^2 + 5x - 6 = 0$

Let $x = 2$ in polynomial $x^2 + 5x - 6$

$x^2 + 5x - 6 = 2^2 + 5 \times 2 - 6$

$= 4 + 10 - 6$

$= 8 \neq 0$

∴ $x = 2$ is not a solution of the equation $x^2 + 5x - 6 = 0$

Solved Example

Ex. $2x^2 - 7x + 6 = 0$ check whether (i) $x = \dfrac{3}{2}$, (ii) $x = -2$ are solutions of the equations.

Solution : (i) Put $x = \dfrac{3}{2}$ in the polynomial $2x^2 - 7x + 6$

$2x^2 - 7x + 6 = 2\left(\dfrac{3}{2}\right)^2 - 7\left(\dfrac{3}{2}\right) + 6$

$$= 2 \times \frac{9}{4} - \frac{21}{2} + 6$$

$$= \frac{9}{2} - \frac{21}{2} + \frac{12}{2} = 0$$

$\therefore x = \dfrac{3}{2}$ is a solution of the equation.

(ii) Let $x = -2$ in $2x^2 - 7x + 6$

$$2x^2 - 7x + 6 = 2(-2)^2 - 7(-2) + 6$$

$$= 2 \times 4 + 14 + 6$$

$$= 28 \neq 0$$

$\therefore x = -2$ is not a solution of the equation.

Activity : If $x = 5$ is a root of equation $kx^2 - 14x - 5 = 0$ then find the value of k by completing the following activity.

Solution : One of the roots of equation $kx^2 - 14x - 5 = 0$ is $\boxed{}$.

$\therefore$ Now Let $x = \boxed{}$ in the equation.

$k\boxed{}^2 - 14\boxed{} - 5 = 0$

$\therefore\quad 25k - 70 - 5 = 0$

$25k - \boxed{} = 0$

$25k = \boxed{}$

$\therefore\quad k = \dfrac{\boxed{}}{\boxed{}} = 3$

Let's remember!

(1) $ax^2 + bx + c = 0$ is the general form of equation where a, b, c are real numbers and 'a' is non zero.

(2) The values of variable which satisfy the equation [or the value for which both the sides of equation are equal] are called solutions or roots of the equation.

Practice Set 2.1

1. Write any two quadratic equations.

2. Decide which of the following are quadratic equations.

(1) $x^2 + 5x - 2 = 0$ (2) $y^2 = 5y - 10$ (3) $y^2 + \dfrac{1}{y} = 2$

(4) $x + \dfrac{1}{x} = -2$ (5) $(m + 2)(m - 5) = 0$ (6) $m^3 + 3m^2 - 2 = 3m^3$

3. Write the following equations in the form $ax^2 + bx + c = 0$, then write the values of a, b, c for each equation.

(1) $2y = 10 - y^2$ (2) $(x - 1)^2 = 2x + 3$ (3) $x^2 + 5x = -(3 - x)$

(4) $3m^2 = 2m^2 - 9$ (5) $P(3 + 6p) = -5$ (6) $x^2 - 9 = 13$

4. Determine whether the values given against each of the quadratic equation are the roots of the equation.

(1) $x^2 + 4x - 5 = 0$, $x = 1, -1$ (2) $2m^2 - 5m = 0$, $m = 2, \dfrac{5}{2}$

5. Find k if $x = 3$ is a root of equation $kx^2 - 10x + 3 = 0$.

6. One of the roots of equation $5m^2 + 2m + k = 0$ is $\dfrac{-7}{5}$. Complete the following activity to find the value of 'k'.

Solution : $\boxed{}$ is a root of quadratic equation $5m^2 + 2m + k = 0$

$\therefore$ Put $m = \boxed{}$ in the equation.

$5 \times \boxed{}^2 + 2 \times \boxed{} + k = 0$

$\boxed{} + \boxed{} + k = 0$

$\boxed{} + k = 0$

$k = \boxed{}$

Let's recall.

Last year you have studied the methods to find the factors of quadratic polynomials like $x^2 - 4x - 5, 2m^2 - 5m, a^2 - 25$. Try the following activity and revise the same.

Activity : Find the factors of the following polynomials.

(1) $x^2 - 4x - 5$

$= \underline{x^2 - 5x + 1x - 5}$

$= x(\ldots) + 1(\ldots)$

$= (\ldots)(\ldots)$

(2) $2m^2 - 5m$

$= \ldots \ldots$

(3) $a^2 - 25$

$= a^2 - 5^2$

$= (\ldots)(\ldots)$

Solutions of a quadratic equation by factorisation

By substituting arbitrary values for the variable and deciding the roots of quadratic equation is a time consuming process. Let us learn to use factorisation method to find the roots of the given quadratic equation.

$x^2 - 4x - 5 = (x - 5)(x + 1)$

$(x - 5)$ and $(x + 1)$ are two linear factors of quadratic polynomial $x^2 - 4x - 5$.

So the quadratic equation obtained from $x^2 - 4x - 5$ can be written as
$(x - 5)(x + 1) = 0$

If product of two numbers is zero then at least one of them is zero.

$\therefore \quad x - 5 = 0$ or $x + 1 = 0$

$\therefore \quad x = 5$ or $x = -1$

$\therefore \quad 5$ and the -1 are the roots of the given quadratic equation.

While solving the equation first we obtained the linear factors. So we call this method as 'factorization method' of solving quadratic equation.

✐✐✐ Solved Examples ✐✐✐

Ex. Solve the following quadratic equations by factorisation.

(1) $m^2 - 14m + 13 = 0$ (2) $3x^2 - x - 10 = 0$

(3) $3y^2 = 15y$ (4) $x^2 = 3$ (5) $6\sqrt{3}x^2 + 7x = \sqrt{3}$

(1) $m^2 - 14m + 13 = 0$

$\therefore m^2 - 13m - 1m + 13 = 0$

$\therefore \overline{m(m - 13)} \; \overline{-1(m - 13)} = 0$

$\therefore (m - 13)(m - 1) = 0$

$\therefore m - 13 = 0$ or $m - 1 = 0$

$\therefore m = 13$ or $m = 1$

$\therefore$ 13 and 1 are the roots of the given quadratic equation.

(2) $3x^2 - x - 10 = 0$

$\therefore \underline{3x^2 - 6x} + \underline{5x - 10} = 0$

$\therefore 3x(x - 2) + 5(x - 2) = 0$

$\therefore (3x + 5)(x - 2) = 0$

$\therefore (3x + 5) = 0$ or $(x - 2) = 0$

$\therefore x = -\dfrac{5}{3}$ or $x = 2$

$\therefore -\dfrac{5}{3}$, and 2 are the roots of the given quadratic equation.

(3) $3y^2 = 15\,y$

    $\therefore\ 3y^2 - 15\,y = 0$

    $\therefore\ 3y\,(y - 5) = 0$

    $\therefore\ \ 3y = 0$ or $(y - 5) = 0$

    $\therefore\ \ y = 0$ or $y = 5$

$\therefore\ $ 0 and 5 are the roots of quadratic

    equation.

(4) $x^2 = 3$

    $\therefore\ x^2 - 3 = 0$

    $\therefore\ x^2 - (\sqrt{3})^2 = 0$

    $\therefore\ (x + \sqrt{3})\,(x - \sqrt{3}) = 0$

    $\therefore (x + \sqrt{3}) = 0$ or $(x - \sqrt{3}) = 0$

    $\therefore\ \ x = -\sqrt{3}$ or $x = \sqrt{3}$

$\therefore\ -\sqrt{3}$ and $\sqrt{3}$ are the roots of given

    quadratic equation.

(5) $6\sqrt{3}\,x^2 + 7x = \sqrt{3}$

$\therefore\ 6\sqrt{3}\,x^2 + 7x - \sqrt{3} = 0$

$\therefore\ 6\sqrt{3}\,x^2 + 9x - 2x - \sqrt{3} = 0$

$\therefore 3\sqrt{3}\,x(2x + \sqrt{3}) - 1\,(2x + \sqrt{3}) = 0$

$\therefore\ (2x + \sqrt{3})\,(3\sqrt{3}\,x - 1) = 0$

$\therefore 2x + \sqrt{3} = 0$ or $3\sqrt{3}\,x - 1 = 0$

$\therefore\ \ 2x = -\sqrt{3}$ or $3\sqrt{3}\,x = 1$

$\therefore\ \ x = -\dfrac{\sqrt{3}}{2}$ or $x = \dfrac{1}{3\sqrt{3}}$

$6\sqrt{3} \times -\sqrt{3}\ = -18$

$\overset{\displaystyle -18}{\underset{\displaystyle 9 \quad\ \ -2}{\diagup\!\!\!\diagdown}}$

$9 = 3\sqrt{3} \times \sqrt{3}$

$\therefore\ -\dfrac{\sqrt{3}}{2}$ and $\dfrac{1}{3\sqrt{3}}$ are the roots of the given quadratic equation.

Practice Set 2.2

1. Solve the following quadratic equations by factorisation.

    (1) $x^2 - 15\,x + 54 = 0$ (2) $x^2 + x - 20 = 0$ (3) $2y^2 + 27\,y + 13 = 0$

    (4) $5m^2 = 22\,m + 15$ (5) $2x^2 - 2\,x + \dfrac{1}{2} = 0$ (6) $6x - \dfrac{2}{x} = 1$

    (7) $\sqrt{2}\,x^2 + 7\,x + 5\sqrt{2} = 0$ to solve this quadratic equation by factorisation,

    complete the following activity.

Solution : $\sqrt{2}\,x^2 + 7\,x + 5\sqrt{2} = 0$

         $\sqrt{2}\,x^2 + \boxed{} + \boxed{} + 5\sqrt{2} = 0$

         $x(.\,.\,.\,.\,.) + \sqrt{2}\,(.\,.\,.\,.\,.) = 0$

$$(\ldots\ldots)(x + \sqrt{2}\,) = 0$$

$$(\ldots\ldots) = 0 \quad \text{or} \quad (x + \sqrt{2}\,) = 0$$

$$\therefore x = \boxed{} \quad \text{or} \quad x = -\sqrt{2}$$

$$\therefore \boxed{} \quad \text{and} \quad -\sqrt{2} \quad \text{are roots of the equation.}$$

(8)★ $3x^2 - 2\sqrt{6}\,x + 2 = 0$ (9) $2m\,(m - 24) = 50$

(10) $25m^2 = 9$ (11) $7m^2 = 21m$ (12) $m^2 - 11 = 0$

 Let's learn.

Solution of a quadratic equation by completing the square

Teacher : Is $x^2 + 10x + 2 = 0$ a quadratic equation or not ?

Yogesh : Yes Sir, because it is in the form $ax^2 + bx + c = 0$, maximum index of the variable x is 2 and 'a' is non zero.

Teacher : Can you solve this equation ?

Yogesh : No Sir, because it is not possible to find the factors of 2 whose sum is 10.

Teacher : Right, so we have to use another method to solve such equations. Let us learn the method.

Let us add a suitable term to $x^2 + 10x$ so that the new expression would be a complete square.

If $x^2 + 10x + k = (x + a)^2$

then $x^2 + 10x + k = x^2 + 2ax + a^2$

$\therefore 10 = 2a$ and $k = a^2$

by equating the coefficients for the variable x and constant term

$\therefore a = 5 \therefore k = a^2 = (5)^2 = 25$

$\therefore x^2 + 10x + 2 = (x + 5)^2 - 25 + 2 = (x + 5)^2 - 23$

Now can you solve the equation $x^2 + 10x + 2 = 0$?

Rehana : Yes Sir, left side of the equation is now difference of two squares and we can factorise it.

$(x + 5)^2 - (\sqrt{23}\,)^2 = 0$

$\therefore (x + 5 + \sqrt{23}\,)(x + 5 - \sqrt{23}\,) = 0$

$\therefore x + 5 + \sqrt{23} = 0$ or $x + 5 - \sqrt{23} = 0$

$\therefore x = -5 - \sqrt{23}$ or $x = -5 + \sqrt{23}$

Hameed : Sir, May I suggest another way ?

$$(x + 5)^2 - (\sqrt{23})^2 = 0$$

$$\therefore (x + 5)^2 = (\sqrt{23})^2$$

$$\therefore x + 5 = \sqrt{23} \text{ or } x + 5 = -\sqrt{23}$$

$$\therefore x = -5 + \sqrt{23} \text{ or } x = -5 - \sqrt{23}$$

✿✿✿ Solved Examples ✿✿✿

Ex. (1) Solve : $5x^2 - 4x - 3 = 0$

Solution : It is convenient to make coefficient of x^2 as 1 and then convert the equation

as the of difference of two squares, so dividing the equation by 5,

we get, $x^2 - \dfrac{4}{5}x - \dfrac{3}{5} = 0$

now if $x^2 - \dfrac{4}{5}x + k = (x - a)^2$ then $x^2 - \dfrac{4}{5}x + k = x^2 - 2ax + a^2$.

compare the terms in $x^2 - \dfrac{4}{5}x$ and $x^2 - 2ax$.

$-2ax = -\dfrac{4}{5}x \quad \therefore a = \dfrac{1}{2} \times \dfrac{4}{5} = \dfrac{2}{5}$

$\therefore k = a^2 = \left(\dfrac{2}{5}\right)^2 = \dfrac{4}{25}$

Now, $x^2 - \dfrac{4}{5}x - \dfrac{3}{5} = 0$

$\therefore x^2 - \dfrac{4}{5}x + \dfrac{4}{25} - \dfrac{4}{25} - \dfrac{3}{5} = 0$

$\therefore \left(x - \dfrac{2}{5}\right)^2 - \left(\dfrac{4}{25} + \dfrac{3}{5}\right) = 0$

$\therefore \left(x - \dfrac{2}{5}\right)^2 - \left(\dfrac{19}{25}\right) = 0$

$\therefore \left(x - \dfrac{2}{5}\right)^2 = \left(\dfrac{19}{25}\right)$

$\therefore x - \dfrac{2}{5} = \dfrac{\sqrt{19}}{5} \text{ or } x - \dfrac{2}{5} = -\dfrac{\sqrt{19}}{5}$

$\therefore x = \dfrac{2}{5} + \dfrac{\sqrt{19}}{5} \text{ or } x = \dfrac{2}{5} - \dfrac{\sqrt{19}}{5}$

$\therefore x = \dfrac{2 + \sqrt{19}}{5} \text{ or } x = \dfrac{2 - \sqrt{19}}{5}$

$\therefore \dfrac{2 + \sqrt{19}}{5} \text{ and } \dfrac{2 - \sqrt{19}}{5}$ are roots of the equation.

> When equation is in the form $x^2 + bx + c = 0$, it can be written as
>
> $$x^2 + bx + \left(\dfrac{b}{2}\right)^2 - \left(\dfrac{b}{2}\right)^2 + c = 0 \text{ that is,}$$
>
> $$\left(x + \dfrac{b}{2}\right)^2 = \left(\dfrac{b}{2}\right)^2 - c$$

Ex. (2) Solve : $x^2 + 8x - 48 = 0$

Method I : Completing the square.

$$x^2 + 8x - 48 = 0$$

$$\therefore x^2 + 8x + 16 - 16 - 48 = 0$$

$$\therefore (x + 4)^2 - 64 = 0$$

$$\therefore (x + 4)^2 = 64$$

$$\therefore x + 4 = 8 \text{ or } x + 4 = -8$$

$$\therefore x = 4 \text{ or } x = -12$$

Method II : Factorisation

$$x^2 + 8x - 48 = 0$$

$$\therefore x^2 + 12x - 4x - 48 = 0$$

$$\therefore x (x + 12) - 4(x + 12) = 0$$

$$\therefore (x + 12) (x - 4) = 0$$

$$\therefore x + 12 = 0 \text{ or } x - 4 = 0$$

$$\therefore x = -12 \text{ or } x = 4$$

Practice Set 2.3

Solve the following quadratic equations by completing the square method.

(1) $x^2 + x - 20 = 0$ (2) $x^2 + 2x - 5 = 0$ (3) $m^2 - 5m = -3$

(4) $9y^2 - 12y + 2 = 0$ (5) $2y^2 + 9y + 10 = 0$ (6) $5x^2 = 4x + 7$

Let's learn.

Formula for solving a quadratic equation

$ax^2 + bx + c$, Divide the polynomial by a ($\because a \neq 0$) to get $x^2 + \dfrac{b}{a}x + \dfrac{c}{a}$.

Let us write the polynomial $x^2 + \dfrac{b}{a}x + \dfrac{c}{a}$ in the form of difference of two square numbers. Now we can obtain roots or solutions of equation $x^2 + \dfrac{b}{a}x + \dfrac{c}{a} = 0$ which is equivalent to $ax^2 + bx + c = 0$.

$$ax^2 + bx + c = 0 \ldots \text{(I)}$$

$$x^2 + \dfrac{b}{a}x + \dfrac{c}{a} = 0 \ldots \ldots \text{dividing both sides by } a$$

$$\therefore x^2 + \dfrac{b}{a}x + \left(\dfrac{b}{2a}\right)^2 - \left(\dfrac{b}{2a}\right)^2 + \dfrac{c}{a} = 0$$

$$\therefore \left(x + \dfrac{b}{2a}\right)^2 - \dfrac{b^2}{4a^2} + \dfrac{c}{a} = 0$$

$$\therefore \left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a^2} = 0 \qquad \therefore \left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$\therefore x + \frac{b}{2a} = \sqrt{\frac{b^2 - 4ac}{4a^2}} \quad \text{or} \quad x + \frac{b}{2a} = -\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$\therefore x = -\frac{b}{2a} + \sqrt{\frac{b^2 - 4ac}{4a^2}} \quad \text{or} \quad x = -\frac{b}{2a} - \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$\therefore x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{or} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

In short the solution is written as $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ and these values are denoted by α, β.

$$\therefore \alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \, , \qquad \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \quad \dots\dots\dots\dots (\text{I})$$

The values of a, b, c from equation $ax^2 + bx + c = 0$ are substituted in $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ and further simplified to obtain the roots of the equation. So

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ is the formula used to solve quadratic equation. Out of the two roots any one can be represented by α and the other by β.

That is, instead (I) we can write $\alpha = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$, $\beta = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ .... (II)

Note that : If $\alpha = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ then $\alpha > \beta$, if $\alpha = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$ then $\alpha < \beta$

✨✨✨ Solved Examples ✨✨✨

Solve quadratic equations using formula.

Ex. (1) $m^2 - 14m + 13 = 0$

Solution : $m^2 - 14m + 13 = 0$ comparing

with $ax^2 + bx + c = 0$

we get $a = 1$, $b = -14$, $c = 13$,

$\therefore b^2 - 4ac = (-14)^2 - 4 \times 1 \times 13$

$\qquad = 196 - 52$

$\qquad = 144$

$m = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\quad = \dfrac{-(-14) \pm \sqrt{144}}{2 \times 1}$

$\quad = \dfrac{14 \pm 12}{2}$

$\therefore m = \dfrac{14 + 12}{2}$ or $m = \dfrac{14 - 12}{2}$

$\therefore m = \dfrac{26}{2}$ or $m = \dfrac{2}{2}$

$\therefore m = 13$ or $m = 1$

$\therefore 13$ and 1 are roots of the equation.

Ex. (2) : $x^2 + 10x + 2 = 0$

Solution : $x^2 + 10x + 2 = 0$ comparing with $ax^2 + bx + c = 0$

we get $a = 1$, $b = 10$, $c = 2$,

$$\therefore b^2 - 4ac = (10)^2 - 4 \times 1 \times 2$$

$$= 100 - 8$$

$$= 92$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-10 \pm \sqrt{92}}{2 \times 1}$$

$$x = \frac{-10 \pm \sqrt{4 \times 23}}{2}$$

$$= \frac{-10 \pm 2\sqrt{23}}{2}$$

$$= \frac{2(-5 \pm \sqrt{23})}{2}$$

$$\therefore \quad x = -5 \pm \sqrt{23}$$

$$\therefore \quad x = -5 + \sqrt{23} \quad \text{or} \quad x = -5 - \sqrt{23}$$

$\therefore$ the roots of the given quadratic equation are $-5 + \sqrt{23}$ and $-5 - \sqrt{23}$.

Ex. (3) : $x^2 - 2x - 3 = 0$

Solution : comparing with $ax^2 + bx + c = 0$

we get $a = 1$, $b = -2$, $c = -3$,

$$\therefore b^2 - 4ac = (-2)^2 - 4 \times 1 \times (-3) = 4 + 12 = 16$$

$$\therefore x = \frac{-(-2) + \sqrt{16}}{2} \quad \text{or} \quad x = \frac{-(-2) - \sqrt{16}}{2}$$

$$= \frac{2 + 4}{2} \quad \text{or} \quad \frac{2 - 4}{2}$$

$$= 3 \quad \text{or} \quad -1$$

For more information :

Let us understand the solution of equation $x^2 - 2x - 3 = 0$ when solved graphically.

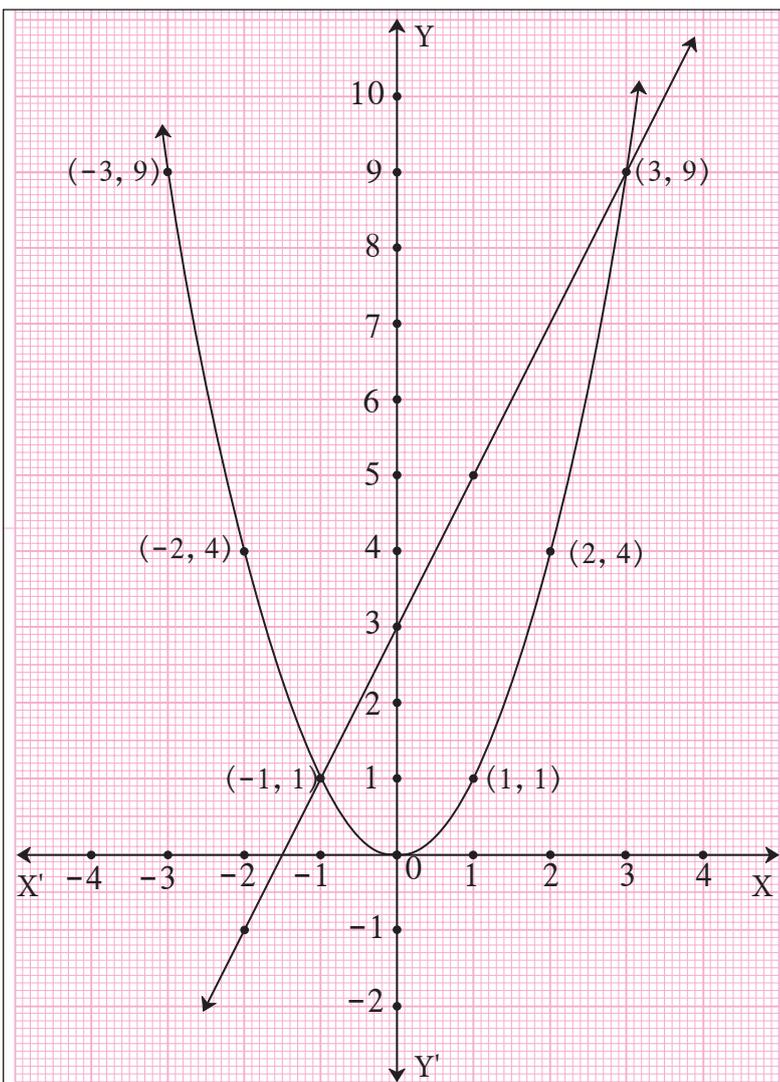$x^2 - 2x - 3 = 0$ $\therefore$ $x^2 = 2x + 3$ The values which satisfy the equation are the roots of the equation.

Let $y = x^2 = 2x + 3$. Let us draw graph of $y = x^2$ and $y = 2x + 3$

$y = x^2$

x	3	2	1	0	−1	−2	−3
y	9	4	1	0	1	4	9

$y = 2x + 3$

x	−1	0	1	−2
y	1	3	5	−1

These graphs intersect each other at (−1, 1) and (3, 9).

$\therefore$ The solutions of $x^2 = 2x + 3$ i.e $x^2 - 2x - 3 = 0$ are $x = -1$ or $x = 3$.

In the adjacent diagram the graphs of equations $y = x^2$ and $y = 2x + 3$ are given. From their points of intersection, observe and understand how you get the solutions of $x^2 = 2x + 3$ i.e solutions of $x^2 - 2x - 3 = 0$.

Ex. (4) $25x^2 + 30x + 9 = 0$

Solution : $25x^2 + 30x + 9 = 0$ comparing the equation with $ax^2 + bx + c = 0$ we get $a = 25$, $b = 30$, $c = 9$,

$\therefore b^2 - 4ac = (30)^2 - 4 \times 25 \times 9$

$= 900 - 900 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-30 \pm \sqrt{0}}{2 \times 25}$

$\therefore x = \dfrac{-30 + 0}{50}$ or $x = \dfrac{-30 - 0}{50}$

$\therefore x = -\dfrac{30}{50}$ or $x = -\dfrac{30}{50}$

$\therefore x = -\dfrac{3}{5}$ or $x = -\dfrac{3}{5}$

that is both the roots are equal.

Also note that $25x^2 + 30x + 9 = 0$ means $(5x + 3)^2 = 0$

Ex. (5) $x^2 + x + 5 = 0$

Solution : $x^2 + x + 5 = 0$ comparing with $ax^2 + bx + c = 0$ we get $a = 1$, $b = 1$, $c = 5$,

$\therefore b^2 - 4ac = (1)^2 - 4 \times 1 \times 5$

$= 1 - 20$

$= -19$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-1 \pm \sqrt{-19}}{2 \times 1}$

$= \dfrac{-1 \pm \sqrt{-19}}{2}$

But $\sqrt{-19}$ is not a real number. Hence roots of the equation are not real.

Activity : Solve the equation $2x^2 + 13x + 15 = 0$ by factorisation method, by completing the square method and by using the formula. Verify that you will get the same roots every time.

Practice Set 2.4

1. Compare the given quadratic equations to the general form and write values of a, b, c.

(1) $x^2 - 7x + 5 = 0$ (2) $2m^2 = 5m - 5$ (3) $y^2 = 7y$

2. Solve using formula.

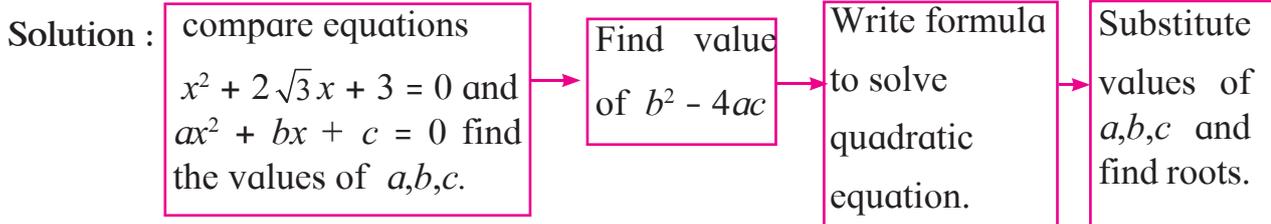(1) $x^2 + 6x + 5 = 0$ (2) $x^2 - 3x - 2 = 0$ (3) $3m^2 + 2m - 7 = 0$

(4) $5m^2 - 4m - 2 = 0$ (5) $y^2 + \dfrac{1}{3}y = 2$ (6) $5x^2 + 13x + 8 = 0$

(3) With the help of the flow chart given below solve the equation $x^2 + 2\sqrt{3}x + 3 = 0$ using the formula.

Solution :

| compare equations $x^2 + 2\sqrt{3}x + 3 = 0$ and $ax^2 + bx + c = 0$ find the values of a,b,c. | → | Find value of $b^2 - 4ac$ | → | Write formula to solve quadratic equation. | → | Substitute values of a,b,c and find roots. |

Let's learn.

Nature of roots of a quadratic equation

You know that $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ are roots of quadratic equation $ax^2 + bx + c = 0$

(1) If $b^2 - 4ac = 0$ then, $x = \dfrac{-b \pm \sqrt{0}}{2a}$ $\therefore x = \dfrac{-b+0}{2a}$ or $x = \dfrac{-b-0}{2a}$

$\therefore$ the roots of the quadratic equation are real and equal.

(2) If $b^2 - 4ac > 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

i.e. $x = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $x = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$

$\therefore$ roots of the quadratic equation are real and unequal.

(3) If $b^2 - 4ac < 0$ then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ are not real numbers $\therefore$ the roots of quadratic equations are not real.

Nature of roots of quadratic equation is determined by the value of $b^2 - 4ac$. $b^2 - 4ac$ is called discriminant of a quadratic equation and is denoted by Greek letter Δ (Delta)

Activity - Fill in the blanks.

Value of discriminant | Nature of roots

(1) 50 →

(2) -30 →

(3) 0 →

Ex. (1) Find the value of the discriminant of the equation $x^2 + 10x - 7 = 0$

Solution : Comparing $x^2 + 10x - 7 = 0$ with $ax^2 + bx + c = 0$.

$$a = 1, \ b = 10, \ c = -7,$$

$$\therefore b^2 - 4ac = 10^2 - 4 \times 1 \times -7$$

$$= 100 + 28$$

$$= 128$$

Ex. (2) Determine nature of roots of the quadratic equations.

(i) $2x^2 - 5x + 7 = 0$

(ii) $x^2 + 2x - 9 = 0$

Solution : Compare $2x^2 - 5x + 7 = 0$ with

$$ax^2 + bx + c = 0$$

$$a = 2, \ b = -5, \ c = 7,$$

$$\therefore b^2 - 4ac = (-5)^2 - 4 \times 2 \times 7$$

$$D = 25 - 56$$

$$D = -31$$

$$\therefore b^2 - 4ac < 0$$

$\therefore$ the roots of the equation are not real.

Solution : Compare $x^2 + 2x - 9 = 0$ with

$$ax^2 + bx + c = 0.$$

$$a = \boxed{}, \ b = 2, \ c = \boxed{},$$

$$\therefore b^2 - 4ac = 2^2 - 4 \times \boxed{} \times \boxed{}$$

$$D = 4 - \boxed{}$$

$$D = 40$$

$$\therefore b^2 - 4ac > 0$$

$\therefore$ the roots of the equation are real and unequal.

Ex. (3) $\sqrt{3}\,x^2 + 2\sqrt{3}\,x + \sqrt{3} = 0$

Solution : Compare $\sqrt{3}\,x^2 + 2\sqrt{3}\,x + \sqrt{3} = 0$ with $ax^2 + bx + c = 0$

We get $a = \sqrt{3}, \ b = 2\sqrt{3}, \ c = \sqrt{3}$,

$$\therefore b^2 - 4ac = (2\sqrt{3})^2 - 4 \times \sqrt{3} \times \sqrt{3}$$

$$= 4 \times 3 - 4 \times 3$$

$$= 12 - 12$$

$$= 0$$

$$\therefore b^2 - 4ac = 0$$

$\therefore$ Roots of the equation are real and equal.

The relation between roots of the quadratic equation and coefficients

α and β are the roots of the equation $ax^2 + bx + c = 0$ then,

$$\alpha + \beta = \frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-b + \sqrt{b^2 - 4ac} - b - \sqrt{b^2 - 4ac}}{2a}$$

$$= -\frac{2b}{2a}$$

$$\therefore \boxed{\alpha + \beta = -\frac{b}{a}}$$

$$\alpha \times \beta = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \times \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{\left(-b + \sqrt{b^2 - 4ac}\right) \times \left(-b - \sqrt{b^2 - 4ac}\right)}{4a^2}$$

$$= \frac{b^2 - \left(b^2 - 4ac\right)}{4a^2}$$

$$= \frac{4ac}{4a^2}$$

$$= \frac{c}{a}$$

$$\therefore \boxed{\alpha \beta = \frac{c}{a}}$$

Activity : Fill in the empty boxes below properly

For $10x^2 + 10x + 1 = 0$,

α + β = ☐ and α × β = ☐

✍✍✍ Solved examples ✍✍✍

Ex. (1) If α and β are the roots of the quadratic equation $2x^2 + 6x - 5 = 0$, then find $(\alpha + \beta)$ and $\alpha \times \beta$.

Solution : Comparing $2x^2 + 6x - 5 = 0$ with $ax^2 + bx + c = 0$.

∴ $a = 2$, $b = 6$, $c = -5$

∴ $\alpha + \beta = -\dfrac{b}{a} = -\dfrac{6}{2} = -3$

and $\alpha \times \beta = \dfrac{c}{a} = \dfrac{-5}{2}$

Ex. (2) The difference between the roots of the equation $x^2 - 13x + k = 0$ is 7 find k.

Solution : Comparing $x^2 - 13x + k = 0$ with $ax^2 + bx + c = 0$

$a = 1$, $b = -13$, $c = k$

Let α and β be the roots of the equation.

$\alpha + \beta = -\dfrac{b}{a} = -\dfrac{(-13)}{1} = 13 \ldots$ (I)

But $\alpha - \beta = 7 \ldots\ldots\ldots$ (given) (II)

$2\alpha = 20 \ldots$ (adding (I) and (II))

$\therefore \alpha = 10$

$\therefore 10 + \beta = 13 \ldots$ (from (I))

$\therefore \beta = 13 - 10$

$\therefore \beta = 3$

But $\alpha \times \beta = \dfrac{c}{a}$

$\therefore 10 \times 3 = \dfrac{k}{1}$

$\therefore k = 30$

Ex. (3) If α and β are the roots of $x^2 + 5x - 1 = 0$ then find –

(i) $\alpha^3 + \beta^3$ (ii) $\alpha^2 + \beta^2$.

Solution : $x^2 + 5x - 1 = 0$

$a = 1$, $b = 5$, $c = -1$

$\alpha + \beta = -\dfrac{b}{a} = \dfrac{-5}{1} = -5$

$\alpha \times \beta = \dfrac{c}{a} = \dfrac{-1}{1} = -1$

(i) $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$= (-5)^3 - 3 \times (-1) \times (-5)$

$= -125 - 15$

$\alpha^3 + \beta^3 = -140$

(ii) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (-5)^2 - 2 \times (-1)$

$= 25 + 2$

$\alpha^2 + \beta^2 = 27$

To obtain a quadratic equation having given roots

Let α and β be the roots of a quadratic equation in variable x

$\therefore x = \alpha$ or $x = \beta$

$\therefore x - \alpha = 0$ or $x - \beta = 0$

$\therefore (x - \alpha)(x - \beta) = 0$

$\therefore x^2 - (\alpha + \beta) x + \alpha\beta = 0$

When two roots of equation are given then quadratic equation can be obtained

as $x^2 -$ (addition of roots) $x +$ product of the roots $= 0$.

Activity (I) : Write the quadratic equation if addition of the roots is 10 and product

of the roots $= 9$

$\therefore$ Quadratic equation : $x^2 - \boxed{} x + \boxed{} = \boxed{}$

Activity (II) : What will be the quadratic equation if $\alpha = 2$, $\beta = 5$

It can be written as $x^2 - (\boxed{} + \boxed{}) x + \boxed{} \times \boxed{} = 0$.

that is $\boxed{} x^2 - \boxed{} x + \boxed{} = 0$.

Note that, if this equation is multiplied by any non zero number, the roots of the
equation are not changed.

ℬℬℬ Solved examples ℬℬℬ

Ex. Obtain the quadratic equation if roots are –3, –7.

Solution : Let $\alpha = -3$ and $\beta = -7$

$\therefore \alpha + \beta = (-3) + (-7) = -10$ and $\alpha \times \beta = (-3) \times (-7) = 21$

$\therefore$ and quadratic equation is, $x^2 - (\alpha + \beta) x + \alpha\beta = 0$

$\therefore x^2 - (-10) x + 21 = 0$

$\therefore x^2 + 10x + 21 = 0$

(1) If α and β are roots of quadratic equation $ax^2 + bx + c = 0$,

(i) $\alpha = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $\beta = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$

(ii) $\alpha + \beta = -\dfrac{b}{a}$ and $\alpha \times \beta = \dfrac{c}{a}$

(2) Nature of roots of quadratic equation $ax^2 + bx + c = 0$ depends on the value of $b^2 - 4ac$. Hence $b^2 - 4ac$ is called discriminant and is denoted by Greek letter Δ.

(3) If $\Delta = 0$ The roots of quadratic equation are real and equal.

If $\Delta > 0$ then the roots of quadratic equation are real and unequal.

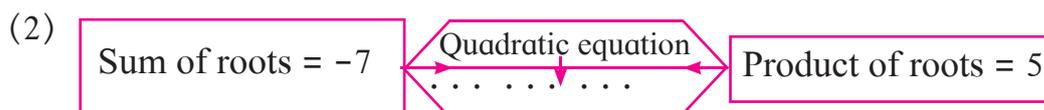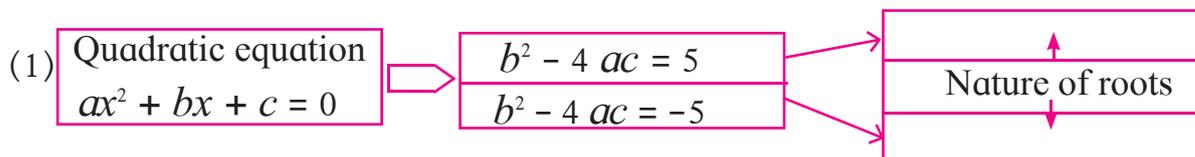If $\Delta < 0$ then the roots of quadratic equation are not real.

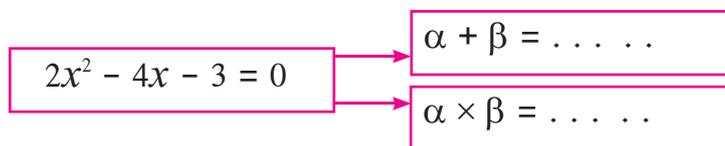(4) The quadratic equation, whose roots are α and β is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

Practice Set 2.5

1. **Activity :** Fill in the gaps and complete.

(1)

Quadratic equation $ax^2 + bx + c = 0$	$b^2 - 4ac = 5$	Nature of roots
	$b^2 - 4ac = -5$	

(2)

Sum of roots = -7	Quadratic equation	Product of roots = 5

(3) If α, β are roots of quadratic equation,

$2x^2 - 4x - 3 = 0$	$\alpha + \beta = \ldots\ldots$
	$\alpha \times \beta = \ldots\ldots$

2. Find the value of discriminant.

(1) $x^2 + 7x - 1 = 0$ (2) $2y^2 - 5y + 10 = 0$ (3) $\sqrt{2}\, x^2 + 4x + 2\sqrt{2} = 0$

3. Determine the nature of roots of the following quadratic equations.

(1) $x^2 - 4x + 4 = 0$ (2) $2y^2 - 7y + 2 = 0$ (3) $m^2 + 2m + 9 = 0$

4. Form the quadratic equation from the roots given below.

   (1) 0 and 4   (2) 3 and –10   (3) $\dfrac{1}{2}$, $-\dfrac{1}{2}$ (4) $2-\sqrt{5}$, $2+\sqrt{5}$

5.★ Sum of the roots of a quadratic equation is double their product. Find k if equation is $x^2 - 4kx + k + 3 = 0$

6.★ α, β are roots of $y^2 - 2y - 7 = 0$ find,

   (1) $\alpha^2 + \beta^2$ (2) $\alpha^3 + \beta^3$

7. The roots of each of the following quadratic equations are real and equal, find k.

   (1) $3y^2 + ky + 12 = 0$ (2) $kx\,(x - 2) + 6 = 0$

Let's learn.

Application of quadratic equation

Quadratic equations are useful in daily life for finding solutions of some practical problems. We are now going to learn the same.

Ex. (1) There is a rectangular onion storehouse in the farm of Mr. Ratnakarrao at Tivasa. The length of rectangular base is more than its breadth by 7 m and diagonal is more than length by 1 m. Find length and breadth of the storehouse.

Solution : Let breadth of the storehouse be x m.

$\therefore$ length $= (x + 7)$ m, diagonal $= x + 7 + 1 = (x + 8)$ m

By Pythagorus theorem

$x^2 + (x + 7)^2 = (x + 8)^2$

$x^2 + x^2 + 14x + 49 = x^2 + 16x + 64$

$\therefore x^2 + 14x - 16x + 49 - 64 = 0$

$\therefore x^2 - 2x - 15 = 0$

$\therefore \underline{x^2 - 5x} + \underline{3x - 15} = 0$

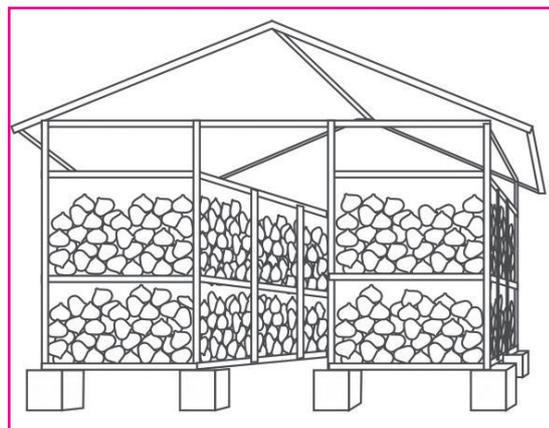$\therefore x(x - 5) + 3\,(x - 5) = 0$

$\therefore (x - 5)\,(x + 3) = 0$

$\therefore x - 5 = 0 \text{ or } x + 3 = 0$

$\therefore x = 5 \text{ or } x = -3$

But length is never negative $\therefore x \neq -3$

$\therefore x = 5$ and $x + 7 = 5 + 7 = 12$

Onion Storehouse (Kandachal)

$\therefore$ Length of the base of storehouse is 12m and breadth is 5m.

Ex. (2) A train travels 360 km with uniform speed. The speed of the train is increased by 5 km/hr, it takes 48 minutes less to cover the same distance. Find the initial speed of the train.

Solution : Let initial speed of the train be x km/hr.

∴ New speed is $(x + 5)$ km/hr.

time to cover 360 km = $\dfrac{\text{distance}}{\text{speed}} = \dfrac{360}{x}$ hours.

New time after increasing speed $= \dfrac{360}{x+5}$ hours.

from given condition

$\dfrac{360}{x+5} = \dfrac{360}{x} - \dfrac{48}{60}$ $\quad$ - - - - - (48 min = $\dfrac{48}{60}$ hrs)

∴ $\dfrac{360}{x} - \dfrac{360}{x+5} = \dfrac{48}{60}$

∴ $\dfrac{1}{x} - \dfrac{1}{x+5} = \dfrac{48}{60 \times 360}$ $\quad$ - - - - - (Dividing both sides by 360)

∴ $\dfrac{x+5-x}{x(x+5)} = \dfrac{4}{5 \times 360}$

∴ $\dfrac{5}{x^2+5x} = \dfrac{1}{5 \times 90}$

∴ $\dfrac{5}{x^2+5x} = \dfrac{1}{450}$

∴ $x^2 + 5x = 2250$

∴ $x^2 + 5x - 2250 = 0$

∴ $x^2 + 50x - 45x - 2250 = 0$ $\qquad$ -2250

∴ $x(x + 50) - 45(x + 50) = 0$ $\qquad$ $+50 \quad -45$

∴ $(x + 50)(x - 45) = 0$

∴ $x + 50 = 0$ or $x - 45 = 0$

∴ $x = -50$ or $x = 45$

But speed is never negative ∴ $x \neq -50$

∴ $x = 45$

∴ Initial speed of the train is 45 km/hr.

1. Product of Pragati's age 2 years ago and 3 years hence is 84. Find her present age.
2. Sum of squares of 2 consecutive natural even numbers is 244; find the numbers.
3. In the orange garden of Mr. Madhusudan there are 150 orange trees. The number of trees in each row is 5 more than that in each column. Find the number of trees in each row and each column with the help of following flow chart.

| No. of trees in a column is x | → | No. of trees in row = . . . | → | Total no. of trees = ... |

| No. of trees in column | ← | Find x | ← | Frame quadratic equation | ← |

↓ no. of trees in a row
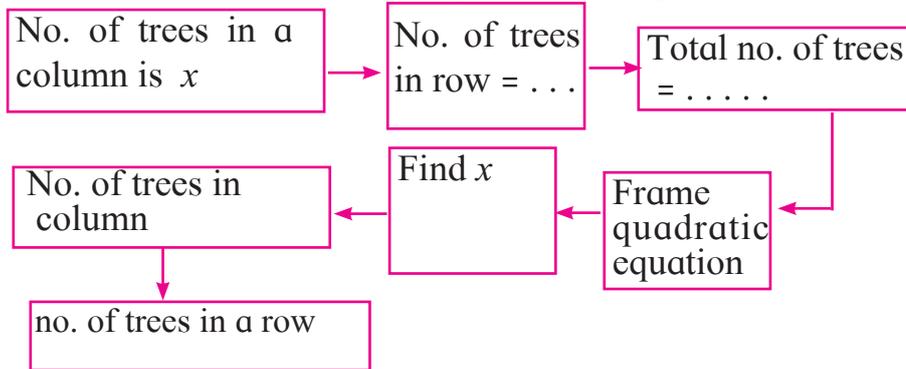
4. Vivek is older than Kishor by 5 years. The sum of the reciprocals of their ages is $\frac{1}{6}$. Find their present ages.
5. Suyash scored 10 marks more in second test than that in the first. 5 times the score of the second test is the same as square of the score in the first test. Find his score in the first test.
6.★ Mr. Kasam runs a small business of making earthen pots. He makes certain number of pots on daily basis. Production cost of each pot is ₹ 40 more than 10 times total number of pots, he makes in one day. If production cost of all pots per day is ₹ 600, find production cost of one pot and number of pots he makes per day.
7.★ Pratik takes 8 hours to travel 36 km downstream and return to the same spot. The speed of boat in still water is 12 km. per hour. Find the speed of water current.
8.★ Pintu takes 6 days more than those of Nishu to complete certain work. If they work together they finish it in 4 days. How many days would it take to complete the work if they work alone.
9.★ If 460 is divided by a natural number, quotient is 6 more than five times the divisor and remainder is 1. Find quotient and diviser.
10.★
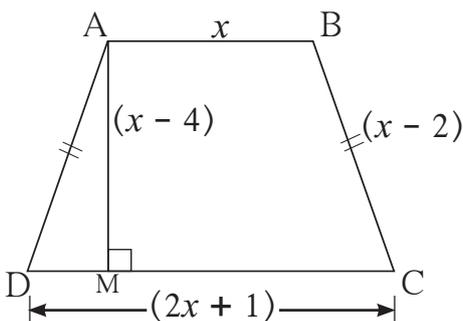
In the adjoining fig. □ABCD is a trapezium AB ‖ CD and its area is 33 cm². From the information given in the figure find the lengths of all sides of the □ABCD. Fill in the empty boxes to get the solution.

Solution : □ABCD is a trapezium.

AB ∥ CD

$A(\square ABCD) = \dfrac{1}{2}(AB + CD) \times \boxed{}$

$33 = \dfrac{1}{2}(x + 2x + 1) \times \boxed{}$

$\therefore \boxed{} = (3x + 1) \times \boxed{}$

$\therefore 3x^2 + \boxed{} - \boxed{} = 0$

$\therefore 3x(\ldots) + 10(\ldots) = 0$

$\therefore (3x + 10)(----) = 0$

$\therefore (3x + 10) = 0$ or $\boxed{} = 0$

$\therefore x = -\dfrac{10}{3}$ or $x = \boxed{}$

But length is never negative.

$\therefore x \neq -\dfrac{10}{3}$ $\quad \therefore x = \boxed{}$

AB = ---, CD = --- , AD = BC = ---

Problem Set - 2

1. Choose the correct answers for the following questions.

   (1) Which one is the quadratic equation ?

   (A) $\dfrac{5}{x} - 3 = x^2$ (B) $x(x + 5) = 2$ (C) $n - 1 = 2n$ (D) $\dfrac{1}{x^2}(x + 2) = x$

   (2) Out of the following equations which one is not a quadratic equation ?

   (A) $x^2 + 4x = 11 + x^2$ (B) $x^2 = 4x$ (C) $5x^2 = 90$ (D) $2x - x^2 = x^2 + 5$

   (3) The roots of $x^2 + kx + k = 0$ are real and equal, find k.

   (A) 0 (B) 4 (C) 0 or 4 (D) 2

   (4) For $\sqrt{2}\,x^2 - 5x + \sqrt{2} = 0$ find the value of the discriminant.

   (A) -5 (B) 17 (C) $\sqrt{2}$ (D) $2\sqrt{2} - 5$

   (5) Which of the following quadratic equations has roots 3, 5 ?

   (A) $x^2 - 15x + 8 = 0$ (B) $x^2 - 8x + 15 = 0$

   (C) $x^2 + 3x + 5 = 0$ (D) $x^2 + 8x - 15 = 0$

   (6) Out of the following equations, find the equation having the sum of its roots –5.

   (A) $3x^2 - 15x + 3 = 0$ (B) $x^2 - 5x + 3 = 0$

   (C) $x^2 + 3x - 5 = 0$ (D) $3x^2 + 15x + 3 = 0$

   (7) $\sqrt{5}\,m^2 - \sqrt{5}\,m + \sqrt{5} = 0$ which of the following statement is true for this given equation ?

   (A) Real and uneual roots (B) Real and equal roots

   (C) Roots are not real (D) Three roots.

   (8) One of the roots of equation $x^2 + mx - 5 = 0$ is 2; find m.

   (A) -2 (B) $-\dfrac{1}{2}$ (C) $\dfrac{1}{2}$ (D) 2

2. Which of the following equations is quadratic ?

   (1) $x^2 + 2x + 11 = 0$ (2) $x^2 - 2x + 5 = x^2$ (3) $(x + 2)^2 = 2x^2$

3. Find the value of discriminant for each of the following equations.

   (1) $2y^2 - y + 2 = 0$ (2) $5m^2 - m = 0$ (3) $\sqrt{5}x^2 - x - \sqrt{5} = 0$

4. One of the roots of quadratic equation $2x^2 + kx - 2 = 0$ is -2, find k.

5. Two roots of quadratic equations are given ; frame the equation.

   (1) 10 and -10 (2) $1-3\sqrt{5}$ and $1+3\sqrt{5}$ (3) 0 and 7

6. Determine the nature of roots for each of the quadratic equation.

   (1) $3x^2 - 5x + 7 = 0$ (2) $\sqrt{3}x^2 + \sqrt{2}x - 2\sqrt{3} = 0$ (3) $m^2 - 2m +1 = 0$

7. Solve the following quadratic equations.

   (1) $\dfrac{1}{x+5} = \dfrac{1}{x^2}$ (2) $x^2 - \dfrac{3x}{10} - \dfrac{1}{10} = 0$ (3) $(2x + 3)^2 = 25$
   (4) $m^2 + 5m + 5 = 0$ (5) $5m^2 + 2m + 1 = 0$ (6) $x^2 - 4x - 3 = 0$

8.★ Find m if $(m - 12)x^2 + 2(m - 12)x + 2 = 0$ has real and equal roots.

9.★ The sum of two roots of a quadratic equation is 5 and sum of their cubes is 35, find the equation.

10.★ Find quadratic equation such that its roots are square of sum of the roots and square of difference of the roots of equation $2x^2 + 2(p + q)x + p^2 + q^2 = 0$

11.★ Mukund possesses ₹ 50 more than what Sagar possesses. The product of the amount they have is 15,000. Find the amount each one has.

12.★ The difference between squares of two numbers is 120. The square of smaller number is twice the greater number. Find the numbers.

13.★ Ranjana wants to distribute 540 oranges among some students. If 30 students were more each would get 3 oranges less. Find the number of students.

14.★ Mr. Dinesh owns an agricultural farm at village Talvel. The length of the farm is 10 meter more than twice the breadth. In order to harvest rain water, he dug a square shaped pond inside the farm. The side of pond is $\dfrac{1}{3}$ of the breadth of the farm. The area of the farm is 20 times the area of the pond. Find the length and breadth of the farm and of the pond

15.★ A tank fills completely in 2 hours if both the taps are open. If only one of the taps is open at the given time, the smaller tap takes 3 hours more than the larger one to fill the tank. How much time does each tap take to fill the tank completely ?

N5AVE9

3 Arithmetic Progression

- Sequence
- Arithmetic Progression
- n^{th} term of an A.P.
- Sum of n terms of an A.P.

Let's learn.

Sequence

We write numbers 1, 2, 3, 4, . . . in an order. In this order we can tell the position of any number. For example, number 13 is at 13th position. The numbers 1, 4, 9, 16, 25, 36, 49, . . . are also written in a particular order. Here $16 = 4^2$ is at 4th position. similarly, $25 = 5^2$ is at the 5th position; $49 = 7^2$ is at the 7th position. In this set of numbers also, place of each number is detremined.

A set of numbers where the numbers are arranged in a definite order, like the natural numbers, is called a **sequence.**

In a sequence a particular number is written at a particular position. If the numbers are written as a_1, a_2, a_3, a_4 . . . then a_1 is first, a_2 is second, . . . and so on. It is clear that a_n is at the nth place. A sequence of the numbers is also represented by alphabets $f_1, f_2, f_3, $ . . . and we find that there is a definite order in which numbers are arranged.

When students stand in a row for drill on the playground they form a sequence. We have experienced that some sequences have a particular pattern.

Complete the given pattern

Pattern	○								
Number of circles	1	3	5	7					

Pattern							
Number of triangles	5	8	11				

Look at the patterns of the numbers. Try to find a rule to obtain the next number from its preceding number. This helps us to write all the next numbers.

See the numbers 2, 11, −6, 0, 5, −37, 8, 2, 61 written in this order.

Here a_1 = 2, a_2 = 11, a_3 = −6, . . .   This list of numbers is also a sequence. But in this case we cannot tell why a particular term is at a particular position ; similarly we cannot tell a definite relation between the consecutive terms.

In general, only those sequences are studied where there is a rule which determines the next term.

For example (1) 4, 8, 12, 16 . . .   (2) 2, 4, 8, 16, 32, . . .

$$(3) \ \frac{1}{5}, \ \frac{1}{10}, \ \frac{1}{15}, \ \frac{1}{20} \ . \ . \ .$$

Terms in a sequence

In a sequence, ordered terms are represented as t_1, t_2, t_3, ...t_n . . . In general sequence is written as $\{t_n\}$. If the sequence is infinite, for every positive integer n, there is a term t_n.

Activity I : Some sequences are given below. Show the positions of the terms by t_1, t_2, t_3, . . .

(1) 9, 15, 21, 27, . . .   Here t_1 = 9, t_2 = 15, t_3 = 21, . . .

(2) 7, 7, 7, 7, . . .   Here t_1 = 7, t_2 = ☐, t_3 = ☐, . . .

(3) −2, −6, −10, −14, . . .   Here t_1 = −2, t_2 = ☐, t_3 = ☐, . . .

Activity II : Some sequences are given below. Check whether there is any rule among the terms. Find the similarity between two sequences.

To check the rule for the terms of the sequence look at the arrangements on the next page, and fill the empty boxes suitably.
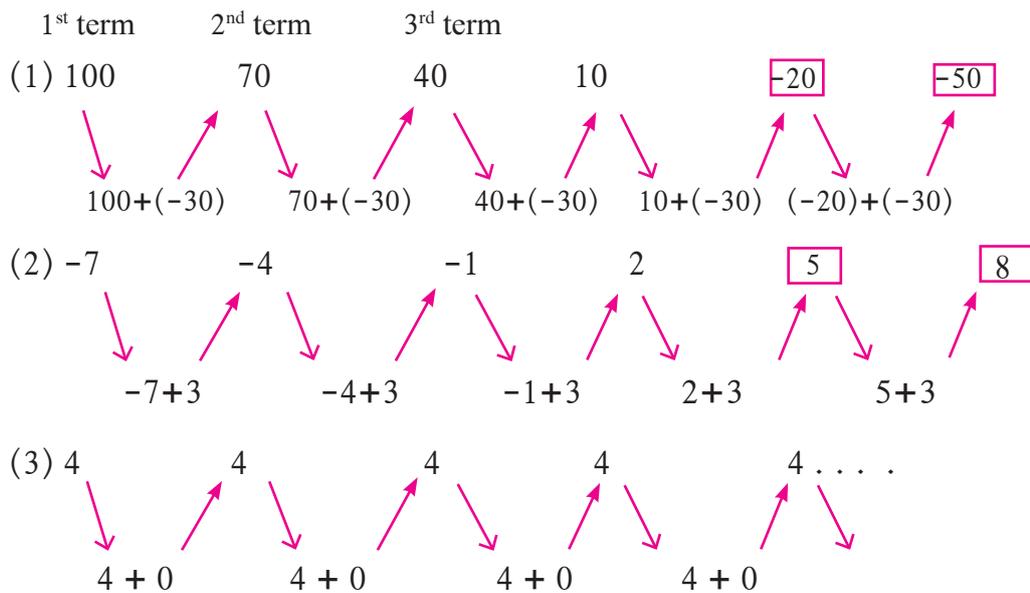
(1) 1, 4, 7, 10, 13, . . .   (2) 6, 12, 18, 24, . . .

(3) 3, 3, 3, 3, . . .   (4) 4, 16, 64, . . .

(5) −1, −1.5, −2, −2.5, . . .   (6) 1^3, 2^3, 3^3, 4^3, . . .

Let's find the relation in these sequences. Let's understand the thought behind it.

(1) 1   4   7   10   ☐   ☐ , . . .

1(+3)   4(+3)   7(+3)   10(+3)   ☐

(2) 6   12   18   24   ☐   ☐ , . . .

6 + 6   12 + 6   18 + 6   ☐   ☐

(3) 3   3   3   3   ☐   ☐ . . .

3 + 0   3 + 0   3 + 0   ☐   ☐

(4) 4   16   64   ☐   ☐   ☐ , . . .

4 × 4   16 × 4   64 × 4   ☐ × 4   ☐ × 4

(5) −1   −1.5   −2   −2.5   ☐ . . .

(−1)+(−0.5)   −1.5 +(−0.5)   −2 + (−0.5)   −2.5 + (−0.5)

(6) 1^3, 2^3, 3^3,   . . .

Here in the sequences (1), (2), (3), (5), the similarity is that next term is obtained by adding a particular number to the previous number. Each ot these sequences is called an Arithmetic Progression.

Sequence (4) is not an arithmetic progression. In this sequence the next term is obtained by mutliplying the previous term by a particular number. This type of sequences is called a Geometric Progression.

Sequence (6) is neither arithmetic progression nor geometric progression.

This year we are going to study arithmetic progression.

Arithmetic Progression

Some sequences are given below. For every sequence write the next three terms.

(1) 100, 70, 40, 10, . . . (2) −7, −4, −1, 2, . . . (3) 4, 4, 4, . . .

In the given sequences, observe how the next term is obtained.

    1ˢᵗ term   2ⁿᵈ term   3ʳᵈ term

(1) 100   70   40   10   −20   −50

    100+(−30)   70+(−30)   40+(−30) 10+(−30) (−20)+(−30)

(2) −7   −4   −1   2   5   8

    −7+3   −4+3   −1+3   2+3   5+3

(3) 4   4   4   4   4 . . . .

    4 + 0   4 + 0   4 + 0   4 + 0

In each sequence above, every term is obtained by adding a particular number in the previous term. The difference between two consecutive terms is constant.

The diference in ex. (i) is negative, in ex. (ii) it is positive and in ex. (iii) it is zero.

If the difference between two consecutive terms is constant then it is called the common difference and is generally denoted by letter d.

In the given sequence if the difference between two consecutive terms $(t_{n+1} - t_n)$ is constant then the sequence is called Arithmetic Progression (A.P.). In this sequence $t_{n+1} - t_n = d$ is the common difference.

In an A.P. if first term is denoted by a and common difference is d then,

$t_1 = a$, $t_2 = a + d$

$t_3 = (a + d) + d = a + 2d$

A.P. having first term as a and common difference d is

$a, (a + d), (a + 2d), (a + 3d), \ldots\ldots$

Let's see some examples of A.P.

Ex.(1) Arifa saved ₹ 100 every month. In one year the total amount saved after every month is as given below.

Month	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
Saving (')	100	200	300	400	500	600	700	800	900	1000	1100	1200

The numbers showing the total saving after every month are in A.P.

Ex. (2) Pranav borrowed ₹ 10000 from his friend and agreed to repay ₹ 1000 per month. So the remaining amount to be paid in every month will be as follows.

No. of month	1	2	3	4	5	...	...	...
Amount to be paid (₹)	10,000	9,000	8,000	7,000	...	2,000	1,000	0

Ex. (3) Consider the table of 5, that is numbers divisible by 5.

5, 10, 15, 20, . . . 50, 55, 60, ... is an inifinite A.P.

Ex (1) and (2) are finite A.P. while (3) is an infinite A.P.

Let's remember!

(1) In a sequence if difference $(t_{n+1} - t_n)$ is constant then the sequence is called an arithmetic progression.

(2) In an A.P. the difference between two consecutive terms is constant and is denoted by d.

(3) Difference d can be positive, negative or zero.

(4) In an A.P. if the first term is a, and common difference is d then the terms in the sequence are a, $(a + d)$, $(a + 2d)$, . . .

Activity : Write one example of finite and infinite A.P. each.

Solved examples

Ex. (1) Which of the following sequences are A.P ? If it is an A.P, find next two terms.

(i) 5, 12, 19, 26, . . .   (ii) 2, −2, −6, −10, . . .

(iii) 1, 1, 2, 2, 3, 3, . . .   (iv) $\frac{3}{2}$, $\frac{1}{2}$, $-\frac{1}{2}$, . . .

Solution : (i) In this sequence 5, 12, 19, 26, . . . ,

First term = t_1 = 5, t_2 = 12, t_3 = 19, . . .

$t_2 - t_1$ = 12 − 5 = 7

$t_3 - t_2$ = 19 − 12 = 7

Here first term is 5 and common difference which is constant is d = 7

∴ This sequence is an A.P.

Next two terms in this A.P. are 26 + 7 = 33 and 33 + 7 = 40.

Next two terms in given A.P. are 33 and 40

(ii) In the sequence 2, -2, -6, -10, . . . ,

$t_1 = 2,$ $t_2 = -2, t_3 = -6,$ $t_4 = -10 . . .$

$t_2 - t_1 = -2 - 2 = -4$

$t_3 - t_2 = -6 - (-2) = -6 + 2 = -4$

$t_4 - t_3 = -10 - (-6) = -10 + 6 = -4$

From this difference between two consecutive terms that is $t_n - t_{n-1} = -4$

$\therefore d = -4,$ which is constant. $\therefore$ It is an A.P.

Next two terms in this A.P. are $(-10) + (-4) = -14$ and $(-14) + (-4) = -18$

(iii) In the sequence 1, 1, 2, 2, 3, 3, . . . ,

$t_1 = 1,$ $t_2 = 1,$ $t_3 = 2,$ $t_4 = 2,$ $t_5 = 3,$ $t_6 = 3 . . .$

$t_2 - t_1 = 1 - 1 = 0$ $t_3 - t_2 = 2 - 1 = 1$

$t_4 - t_3 = 2 - 2 = 0$ $t_3 - t_2 \neq t_2 - t_1$

In this sequence difference between two consecutive terms is not constant.

$\therefore$ This sequence is not an A.P.

(iv) In the sequence $\dfrac{3}{2}, \dfrac{1}{2}, -\dfrac{1}{2}, -\dfrac{3}{2}, \cdots ,$

$t_1 = \dfrac{3}{2},$ $t_2 = \dfrac{1}{2}, t_3 = -\dfrac{1}{2},$ $t_4 = -\dfrac{3}{2} \cdots$

$t_2 - t_1 = \dfrac{1}{2} - \dfrac{3}{2} = -\dfrac{2}{2} = -1$

$t_3 - t_2 = -\dfrac{1}{2} - \dfrac{1}{2} = -\dfrac{2}{2} = -1$

$t_4 - t_3 = -\dfrac{3}{2} - (-\dfrac{1}{2}) = -\dfrac{3}{2} + \dfrac{1}{2} = -\dfrac{2}{2} = -1$

Here the common difference $d = -1$ which is constant.

$\therefore$ Given sequence is an A.P. Let's find next two terms of this A.P.

$-\dfrac{3}{2} - 1 = -\dfrac{5}{2},$ $\dfrac{5}{2} - 1 = -\dfrac{7}{2}$

$\therefore$ Next two terms are $-\dfrac{5}{2}$ and $-\dfrac{7}{2}$

Ex. (2) The first term a and common difference d are given. Find first four terms of A.P.

    (i) $a = -3, d = 4$ (ii) $a = 200, d = 7$

    (iii) $a = -1, d = -\dfrac{1}{2}$ (iv) $a = 8, d = -5$

Solution : (i) Given $a = -3, d = 4$

    $t_1 = -3$

    $t_2 = t_1 + d = -3 + 4 = 1$

    $t_3 = t_2 + d = 1 + 4 = 5$

    $t_4 = t_3 + d = 5 + 4 = 9$

    $\therefore$ A.P. is $= -3, 1, 5, 9, \ldots$

(ii) Given $a = 200, d = 7$

    $a = t_1 = 200$

    $t_2 = t_1 + d = 200 + 7 = 207$

    $t_3 = t_2 + d = 207 + 7 = 214$

    $t_4 = t_3 + d = 214 + 7 = 221$

    $\therefore$ A.P. is $= 200, 207, 214, 221, \ldots$

(iii) $a = -1, d = -\dfrac{1}{2}$

    $a = t_1 = -1$

    $t_2 = t_1 + d = -1 + \left(-\dfrac{1}{2}\right) = -\dfrac{3}{2}$

    $t_3 = t_2 + d = -\dfrac{3}{2} + \left(-\dfrac{1}{2}\right) = -\dfrac{4}{2} = -2$

    $t_4 = t_3 + d = -2 + \left(-\dfrac{1}{2}\right)$

    $= -2 - \dfrac{1}{2} = -\dfrac{5}{2}$

$\therefore$ A.P. is $= -1, -\dfrac{3}{2}, -2, -\dfrac{5}{2}, \ldots$

(iv) $a = 8, d = -5$

    $a = t_1 = 8$

    $t_2 = t_1 + d = 8 + (-5) = 3$

    $t_3 = t_2 + d = 3 + (-5) = -2$

    $t_4 = t_3 + d = -2 + (-5) = -7$

    $8, 3, -2, -7, \ldots$

    $\therefore$ A.P. is $= 8, 3, -2, -7, \ldots$

Practice Set 3.1

1. Which of the following sequences are A.P. ? If they are A.P. find the common difference .

    (1) $2, 4, 6, 8, \ldots$ (2) $2, \dfrac{5}{2}, 3, \dfrac{7}{3}, \ldots$ (3) $-10, -6, -2, 2, \ldots$

    (4) $0.3, 0.33, .0333, \ldots$ (5) $0, -4, -8, -12, \ldots$ (6) $-\dfrac{1}{5}, -\dfrac{1}{5}, -\dfrac{1}{5}, \ldots$

    (7) $3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}, \ldots$ (8) $127, 132, 137, \ldots$

2. Write an A.P. whose first term is a and common difference is d in each of the following.

    (1) $a = 10, d = 5$ (2) $a = -3, d = 0$ (3) $a = -7, d = \dfrac{1}{2}$

    (4) $a = -1.25, d = 3$ (5) $a = 6, d = -3$ (6) $a = -19, d = -4$

3. Find the first term and common difference for each of the A.P.

(1) 5, 1, −3, −7, . . .   (2) 0.6, 0.9, 1.2, 1.5, . . .

(3) 127, 135, 143, 151, . . .   (4) $\dfrac{1}{4}, \dfrac{3}{4}, \dfrac{5}{4}, \dfrac{7}{4}, \ldots$

Let's think.

• Is 5, 8, 11, 14, . . . an A.P. ? If so then what will be the 100th term?
Check whether 92 is in this A.P.? Is number 61 in this A.P.?

Let's learn.

n^{th} term of an A. P.

In the sequence 5, 8, 11, 14, . . . the difference between two consecutive terms is 3. Hence, this sequence is an A.P.

Here the first term is 5. If 3 is added to 5 we get the second term 8. Similarly to find 100th term what should be done?

	First term	Second term	Third term . . .
Number	5,	5 + 3 = 8	8 + 3 = 11 . . .

In this way reaching upto 100th term will be time consuming. Let's see if we can find any formula for it.

5	8	11	14	. . .	. . .	. . .	. . .
5	$5 + 1 \times 3$	$5 + 2 \times 3$	$5 + 3 \times 3$	. . .	$5 + (n - 1) \times 3$	$5 + n \times 3$	. . .
1st term	2nd term	3rd term	4th term	. . .	n^{th} term	$(n + 1)^{th}$ term	. . .
t_1	t_2	t_3	t_4		t_n	t_{n+1}	

Generally in the A.P. $t_1, t_2, t_3, \ldots$ If first term is a and common difference is d,

$$t_1 = a$$

$$t_2 = t_1 + d = a + d = a + (2 - 1)\,d$$

$$t_3 = t_2 + d = a + d + d = a + 2d = a + (3 - 1)d$$

$$t_4 = t_3 + d = a + 2d + d = a + 3d = a + (4 - 1)d$$

We get $t_n = a + (n - 1)\,d.$

Using the above formula we can find the 100th term of the A.P. 5, 8, 11, 14, . . .

Here $a = 5$ $d = 3$

$$t_n = a + (n - 1)d$$

$$\therefore\ t_{100} = 5 + (100 - 1) \times 3$$

$$= 5 + 99 \times 3$$

$$= 5 + 297$$

$$t_{100} = 302$$

100th tem of this A.P. is 302.

Let's check whether 61 is in this A.P. To find the answer we use the same formula.

$$t_n = a + (n - 1)d$$

$$t_n = 5 + (n - 1) \times 3$$

If 61 is n^{th} term means t_n, then

$$\therefore\ 61 = 5 + 3n - 3$$

$$= 3n + 2$$

$$\therefore\ 3n = 59$$

$$\therefore\ n = \frac{59}{3}$$

But then, n is not a natural number.

$\therefore$ 61 is not in this A.P.

Let's think.

Kabir's mother keeps a record of his height on each birthday. When he was one year old, his height was 70 cm, at 2 years he was 80 cm tall and 3 years he was 90 cm tall. His aunt Meera was studying in the 10th class. She said, ''it seems like Kabir's height grows in Arithmetic Progression''. Assuming this, she calculated how tall Kabir will be at the age of 15 years when he is in the 10th ! She was shocked to find it. You too assume that Kabir grows in A.P. and find out his height at the age of 15 years.

Ex. (1) Find t_n for following A.P. and then find 30th term of A.P.

3, 8, 13, 18, . . .

Solution : Given A.P. 3, 8, 13, 18, . . .

Here $t_1 = 3$, $t_2 = 8$, $t_3 = 13$, $t_4 = 18$, . . .

$d = t_2 - t_1 = 8 - 3 = 5$

We know that $t_n = a + (n - 1)d$

$\therefore t_n = 3 + (n - 1) \times 5$ $\because a = 3$, $d = 5$

$\therefore t_n = 3 + 5n - 5$

$\therefore t_n = 5n - 2$

$\therefore 30^{th}$ term $= t_{30} = 5 \times 30 - 2$

$= 150 - 2 \quad = 148$

Ex. (2) Which term of the following A.P. is 560?

2, 11, 20, 29, . . .

Solution : Given A.P. 2, 11, 20, 29, . .

Here $a = 2$, $d = 11 - 2 = 9$

n^{th} term of this A.P. is 560.

$t_n = a + (n - 1)d$

$\therefore 560 = 2 + (n - 1) \times 9$

$= 2 + 9n - 9$

$\therefore 9n = 567$

$\therefore n = \dfrac{567}{9} = 63$

$\therefore 63^{rd}$ term of given A.P. is 560.

Ex. (3) Check whether 301 is in the sequence 5, 11, 17, 23, . . . ?

Solution : In the sequence 5, 11, 17, 23, . . .

$t_1 = 5$, $t_2 = 11$, $t_3 = 17$, $t_4 = 23$, . . .

$t_2 - t_1 = 11 - 5 = 6$

$t_3 - t_2 = 17 - 11 = 6$

$\therefore$ This sequence is an A.P.

First term $a = 5$ and $d = 6$

If 301 is n^{th} term, then.

$t_n = a + (n - 1)d = 301$

$\therefore 301 = 5 + (n - 1) \times 6$

$= 5 + 6n - 6$

$\therefore 6n = 301 + 1 = 302$

$\therefore n = \dfrac{302}{6}$. But it is not an integer.

$\therefore 301$ is not in the given sequence.

Ex. (4) How many two digit numbers are divisible by 4?

Solution : List of two digit numbers divisible by 4 is

12, 16, 20, 24, . . . , 96.

Let's find how many such numbers are there.

$t_n = 96$, $\quad a = 12$, $\quad d = 4$

From this we will find the value of n.

$t_n = 96$, $\therefore$ By formula,

$96 = 12 + (n - 1) \times 4$

$= 12 + 4n - 4$

$\therefore 4n = 88$

$\therefore n = 22$

$\therefore$ There are 22 two digit numbers divisible by 4.

Ex. (5) – The 10th term and the 18th term of an A.P. are 25 and 41 respectively then find 38th term of that A.P., similarly if n^{th} term is 99. Find the value of n.

Solution : In the given A.P. $t_{10} = 25$ and $t_{18} = 41$.

We know that, $t_n = a + (n - 1)d$

$\therefore$ $t_{10} = a + (10 - 1)d$

$\therefore$ $25 = a + 9d$ ... (I)

Similarly $t_{18} = a + (18 - 1)d$

$\therefore$ $41 = a + 17d$ ... (II)

$25 = a + 9d$ ... From (I) .

$a = 25 - 9d$.

Substituting this value in equation II.

$\therefore$ Equation (II) $a + 17d = 41$

$\therefore$ $25 - 9d + 17d = 41$

$8d = 41 - 25 = 16$

$d = 2$

Substituting $d = 2$ in equation I.

$a + 9d = 25$

$\therefore$ $a + 9 \times 2 = 25$

$\therefore$ $a + 18 = 25$

$\therefore$ $a = 7$

Now, $t_n = a + (n - 1)d$

$\therefore$ $t_{38} = 7 + (38 - 1) \times 2$

$t_{38} = 7 + 37 \times 2$

$t_{38} = 7 + 74$

$t_{38} = 81$

If n^{th} term is 99, then to find value of n.

$t_n = a + (n - 1)d$

$99 = 7 + (n - 1) \times 2$

$99 = 7 + 2n - 2$

$99 = 5 + 2n$

$\therefore$ $2n = 94$

$\therefore$ $n = 47$

$\therefore$ In the given progression 38th term is 81 and 99 is the 47th term.

1. Write the correct number in the given boxes from the following A. P.

    (i) 1, 8, 15, 22, . . .

    Here $a = \boxed{}$, $t_1 = \boxed{}$, $t_2 = \boxed{}$, $t_3 = \boxed{}$,

    $t_2 - t_1 = \boxed{} - \boxed{} = \boxed{}$

    $t_3 - t_2 = \boxed{} - \boxed{} = \boxed{} \therefore d = \boxed{}$

    (ii) 3, 6, 9, 12, . . .

    Here $t_1 = \boxed{}$, $t_2 = \boxed{}$, $t_3 = \boxed{}$, $t_4 = \boxed{}$,

    $t_2 - t_1 = \boxed{}$, $t_3 - t_2 = \boxed{} \qquad \therefore d = \boxed{}$

    (iii) −3, −8, −13, −18, . . .

    Here $t_3 = \boxed{}$, $t_2 = \boxed{}$, $t_4 = \boxed{}$, $t_1 = \boxed{}$,

    $t_2 - t_1 = \boxed{}$, $t_3 - t_2 = \boxed{} \qquad \therefore a = \boxed{}$, $d = \boxed{}$

    (iv) 70, 60, 50, 40, . . .

    Here $t_1 = \boxed{}$, $t_2 = \boxed{}$, $t_3 = \boxed{}$, . . .

    $\therefore a = \boxed{}$, $d = \boxed{}$

2. Decide whether following sequence is an A.P., if so find the 20th term of the progression.

    −12, −5, 2, 9, 16, 23, 30, . . .

3. Given Arithmetic Progression 12, 16, 20, 24, . . . Find the 24th term of this progression.

4. Find the 19th term of the following A.P.

    7, 13, 19, 25, . . .

5. Find the 27th term of the following A.P.

    9, 4, −1, −6, −11, . . .

6. Find how many three digit natural numbers are divisible by 5.

7. The 11th term and the 21st term of an A.P. are 16 and 29 respectively, then find the 41th term of that A.P.

8. 11, 8, 5, 2, . . . In this A.P. which term is number −151?

9. In the natural numbers from 10 to 250, how many are divisible by 4?

10. In an A.P. 17th term is 7 more than its 10th term. Find the common difference.

The Wise Teacher

Once upon a time, there lived a king. He appointed two teachers Tara and Meera to teach horse riding for one year to his children Yashwantraje and Geetadevi. He asked both of them how much salary they wanted.

Tara said, ''Give me 100 gold coins in first month and every month increase the amount by 100 gold coins.'' Meera said, ''Give me 10 gold coins in the first month and every month just double the amount of the previous month.''

The king agreed. After three months Yashwantraje said to his sister, ''My teacher is smarter than your teacher as she had asked for more money.'' Geetadevi said, ''I also thought the same, I asked Meeratai about it. She only smiled and said compare the salaries after 8 months. I calculated their 9 months salaries. You can also check.''

Months	1	2	3	4	5	6	7	8	9	10	11	12
Tara's salary	100	200	300	400	500	600	700	800	900	–	–	–
Meera's salary	10	20	40	80	160	320	640	1280	2560	–	–	–

Complete the above table.

Tara's salary 100, 200, 300, 400, . . . is in A.P.

$t_1 = 100$, $t_2 = 200$, $t_3 = 300$,. . . $t_2 - t_1 = 100 = d$

Common difference is 100.

Meera's salary 10, 20, 40, 80, . . . is not an A.P. Reason is $20 - 10 = 10$, $40 - 20 = 20$, $80 - 40 = 40$ So the common difference is not constant.

But here each term is double the preceding term.

Here $\dfrac{t_2}{t_1} = \dfrac{20}{10} = 2$, $\dfrac{t_3}{t_2} = \dfrac{40}{20} = 2$, $\dfrac{t_4}{t_3} = \dfrac{80}{40} = 2$

$\therefore \dfrac{t_{n+1}}{t_n}$ The ratio of a term and its preceding term is constant. This type of progression is called a geometric progression. Notice that if ratio $\dfrac{t_{n+1}}{t_n}$ is greater than 1, then geometric progression will increase faster than arithmetic progression.

If the ratio is smaller than 1, note how the geometric progression changes.

This year we are going to study Arithmetic Progression only. We have seen how to find the nth term of an A.P. Now we are going to see how to find the sum of the first n terms.

Quick Addition

Three hundred years ago there was a single teacher school in Germany. The teacher was Buttner and he had an assistant Johann Martin Bortels. He used to teach alphabets to the children and sharpen their pencils. Buttner was a strict teacher. One day he wanted to do some work and wanted peace in the class, so he tried to occupy all students with a lengthy addition. They were asked to add all intergers from 1 to 100. In few minutes one slate was slammed on the floor. He looked at Carl Gauss and asked, "I asked you to add all integers from 1 to 100. Why did you keep the slate down? Don't you want to do it ?"

Carl Gauss said, "I have done the addition."

The teacher asked, "How did you do it so quickly? You wouldn't have written all the numbers ! What is the answer ?"

Carl Gauss said, "Five thousand fifty"

Teacher was so surprised and asked him, 'How do you find the answer?"

Carl Gauss explained his quick addition method:

| Nos. in increasing order | 1 | 2 | 3 | 4 | . . . . .100 |
| Nos. in decreasing order | 100 | 99 | 98 | 97 | . . . . .1 |
| Sum | 101 | 101 | 101 | 101 | 101 |

The sum of each pair is 101. This sum occurs 100 times so 101×100 is the product needed. It is 10100. In this 1 to 100 are counted two times. Therefore, half of 10100 is 5050 and sum of 1 to 100 is 5050. The teacher appreciated his work.

Now using this method of Gauss, let's find sum of n terms of an A.P.

<div style="border:pink">

Johann Friedrich Carl Gauss

30th April 1777 – 23rd February 1855.

Carl Gauss was a great German mathematician, He was born in Braunschweig, he was the only son of uneducated parents. He showed a glimpse of his intelligence in Buttner's school. After some years, Buttner's helper, Johann Martin Bartels and Gauss became friends. Together, they published a book on Algebra. Bartels made the other people realise the extra ordinary intelligence of Gauss.

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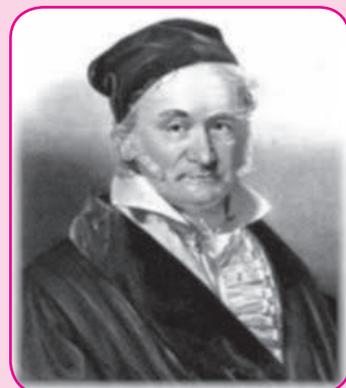
Sum of first n terms of an A. P.

Arithmetic Progression $a, a + d, a + 2d, a + 3d, \ldots a +(n - 1)d$

In this progression a is the first term and d is the common difference. Let's write the sum of first n terms as S_n.

$S_n = \quad [a] \qquad + \quad [a + d] \qquad + \ldots + [a+(n-2)d] + [a+(n-1)d]$

Reversing the terms and rewritting the expression again,

$S_n = [a+(n-1)d] \qquad + [a+(n-2)d] \qquad + \ldots + [a + d] + \quad [a]$

On adding,

$2S_n = [a+a+(n-1)d] + [a + d+a+(n-2)d]+ \ldots + [a+(n-2)d+ a + d]+ [a+(n-1)d+a]$

$2S_n = [2a+(n-1)d] \quad + [2a+(n-1)d] \qquad + \ldots + [2a+(n-1)d] \ldots n$ times.

$\therefore \ 2S_n = n [2a+(n-1)d]$

$\therefore \ S_n = \dfrac{n}{2} [2a+(n-1)d] \quad$ or $\qquad S_n = na + \dfrac{n(n-1)}{2} d$

Ex. Let's find the sum of first 100 terms of A.P. 14, 16, 18, ...

Here $a = 14, \quad d = 2, \qquad n = 100$

$$S_n = \dfrac{n}{2} [2a+(n-1)d]$$

$$\therefore \ S_{100} = \dfrac{100}{2} [2 \times 14+(100-1) \times 2]$$

$$= 50 [28 + 198]$$

$$= 50 \times 226 \ = 11300$$

$\therefore$ Sum of first 100 terms of given A.P. is 11,300

Let's remember!

For the given Arithmetic Progression, if first term is a and common diffrence is d then

$$t_n = [a+(n-1)d]$$

$$S_n = \dfrac{n}{2} [2a+(n-1)d]= na + \dfrac{n(n-1)}{2} d$$

Let's find one more formula for sum of first n terms.

In the A.P. $a, a + d, a + 2d, a + 3d, \ldots a + (n - 1)d$

First term $= t_1 = a$ and n^{th} term is $[a + (n - 1)d]$

Now $S_n = \dfrac{n}{2}\left[\underline{a + a + (n-1)d}\right]$

$\therefore \quad S_n = \dfrac{n}{2}[t_1 + t_n] = \dfrac{n}{2}$ [First term + last term]

✍✍✍ Solved examples ✍✍✍

Ex. (1) Find the sum of first n natural numbers.

Solution : First n natural numbers are $1, 2, 3, \ldots, n$.

Here $a = 1$, $d = 1$, n^{th} term $= n$

$\therefore S_n = 1 + 2 + 3 + \ldots + n$

$S_n = \dfrac{n}{2}$ [First term + last term] $\ldots$ (by the formula)

$= \dfrac{n}{2}[1 + n]$

$= \dfrac{n(n+1)}{2}$

$\therefore$ Sum of first n natural number is $\dfrac{n(n+1)}{2}$.

Ex. (2) Find the sum of first n even natural numbers.

Solution : First n even natural numbers are $2, 4, 6, 8, \ldots, 2n$.

$t_1 =$ First term $= 2$, $t_n =$ last term $= 2n$

Method I	Method II	Method III
$= \dfrac{n}{2}[t_1 + t_n]$	$S_n = 2 + 4 + 6 \ldots + 2n$	$S_n = \dfrac{n}{2}[2a + (n-1)d]$
$= \dfrac{n}{2}[2 + 2n]$	$= 2(1 + 2 + 3 + \ldots + n)$	$= \dfrac{n}{2}[2 \times 2 + (n-1)2]$
$= \dfrac{n}{2} \times 2(1 + n)$	$= \dfrac{2[n(n+1)]}{2}$	$= \dfrac{n}{2}[4 + 2n - 2]$
$= n(1 + n)$	$= n(1 + n)$	$= \dfrac{n}{2}[2 + 2n]$
		$= \dfrac{n}{2} \times 2(1 + n)$
		$= n(1 + n)$

$\therefore$ Sum of first n even natural numbers is $n(1 + n)$.

Ex. (3) Find the sum of first n odd natural numbers.

Solution : First n natural numbers

$\qquad$ $1, 3, 5, 7, \ldots, (2n - 1)$.

$\qquad$ $a = t_1 = 1$ and $t_n = (2n - 1)$, $d = 2$

Method I	Method II	Method III
$S_n = \dfrac{n}{2}\,[t_1 + t_n]$	$S_n = \dfrac{n}{2}\,[2a+(n-1)d]$	$1 + 3 + \ldots\ 2n-1$
$\quad = \dfrac{n}{2}\,[1 + (2n - 1)]$	$\quad = \dfrac{n}{2}\,[2 \times 1 +(n-1) \times 2]$	$= (1 + 2 + 3 + \ldots + 2n)$
$\quad = \dfrac{n}{2}\,[1 + 2n - 1]$	$\quad = \dfrac{n}{2}\,[2 + 2n - 2]$	$\quad\ - (2 + 4 + 6 + \ldots + 2n)$
$\quad = \dfrac{n}{2} \times 2n$	$\quad = \dfrac{n}{2} \times 2n$	$= \dfrac{2n(2n+1)}{2} - \dfrac{2n(n+1)}{2}$
$\quad = n^2$	$\quad = n^2$	$= (2n^2 + n) - (n^2 + n)$
		$= n^2$

Ex. (4) Find the sum of all odd numbers from 1 to 150.

Solution : 1 to 150 all odd numbers are $1, 3, 5, 7, \ldots , 149$.

$\qquad$ Which is an A.P.

$\qquad$ Here $a = 1$ and $d = 2$. First let's find how many odd numbers are there from 1 to 150, so find the value of n, if $t_n = 149$

$\qquad$ $t_n = a +(n - 1)d$

$\qquad$ $149 = 1 +(n - 1)2$ $\qquad\qquad \therefore\ 149 = 1 + 2n - 2$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad \therefore\ n = 75$

$\qquad$ Now let's find the sum of these 75 numbers $\qquad 1 + 3 + 5 + \ldots + 149$.

$\qquad$ $a = 1$ and $d = 2$, $n = 75$

Method I $\quad S_n = \dfrac{n}{2}\,[2a+(n-1)d]$

$\qquad\qquad S_n = \boxed{}$

$\qquad\qquad S_n = \boxed{} \times \boxed{}$

$\qquad\qquad S_n = \boxed{}$

Method II $\quad S_n = \dfrac{n}{2}\,[t_1 + t_n]$

$\qquad\qquad S_n = \dfrac{75}{2}\,[1 + 149]$

$\qquad\qquad S_n = \boxed{} \times \boxed{}$

$\qquad\qquad S_n = \boxed{}$

1. First term and common difference of an A.P. are 6 and 3 respectively ; find S_{27}.

$a = 6, \ d = 3, S_{27} = \ ?$

$S_n = \dfrac{n}{2} \ [\square + (n-1) \ d]$

$S_{27} = \dfrac{27}{2} \ [12 + (27-1) \ \square]$

$\quad = \dfrac{27}{2} \ \times \square$

$\quad = 27 \times 45 \ = \boxed{}$

2. Find the sum of first 123 even natural numbers.

3. Find the sum of all even numbers between 1 and 350.

4. In an A.P. 19th term is 52 and 38th term is 128, find sum of first 56 terms.

5. Complete the following activity to find the sum of natural numbers between 1 and 140 which are divisible by 4.

$\boxed{\text{Between 1 and 140, natural numbers divisible by 4}}$

$\boxed{4, \ 8, \ . \ . \ . \ . \ . \ . \ . \ . \ . \ , \ 136}$

$\boxed{\text{How many numbers ? } \therefore \ n = \square}$

$\boxed{n = \square \ , \ a = \square \ , \ d = \square}$

$\boxed{t_n = a + (n-1)d}$

$\boxed{136 = \square + (n-1) \times \square}$

$n = \square \qquad S_n = \dfrac{n}{2} \ [2a + (n-1)d]$

$\boxed{S_\square = \dfrac{\square}{2} \ [\qquad\qquad]} \qquad = \boxed{}$

Sum of numbers from 1 to 140, which are divisible by 4 = $\boxed{}$

6. ⋆ Sum of first 55 terms in an A.P. is 3300, find its 28th term.

7. In an A.P. sum of three consecutive terms is 27 and their product is 504, find the terms. (Assume that three consecutive terms in A.P. are $a - d$, a, $a + d$.)

8. Find four consecutive terms in an A.P. whose sum is 12 and sum of 3rd and 4th term is 14.

   (Assume the four consecutive terms in A.P. are $a - d$, a, $a + d$, $a + 2d$.)

9. If the 9th term of an A.P. is zero then show that the 29th term is twice the 19th term.

Let's learn.

Application of A.P.

Ex. (1) A mixer manufacturing company manufactured 600 mixers in 3rd year and in 7th year they manufactured 700 mixers. If every year there is same growth in the production of mixers then find (i) Production in the first year (ii) Production in 10th year (iii) Total production in first seven years.

Solution : Addition in the number of mixers manufactured by the company per year is constant therefore the number of production in successive years is in A.P.

(i) Let's assume that company manufactured t_n mixers in the nth year then as per given information,

$t_3 = 600$, $t_7 = 700$

We know that $t_n = a + (n-1)d$

$t_3 = a + (3-1)d$

$a + 2d = 600$. . . (I)

$t_7 = a + (7-1)d$

$t_7 = a + 6d = 700$

$a + 2d = 600$ $\therefore$ Substituting $a = 600 - 2d$ in equation (II),

$600 - 2d + 6d = 700$

$4d = 100$ $\therefore$ $d = 25$

$a + 2d = 600$ $\therefore$ $a + 2 \times 25 = 600$

$a + 50 = 600$ $\therefore$ $a = 550$

$\therefore$ Production in first year was 550.

(ii) $t_n = a + (n-1)d$

$t_{10} = 550 + (10-1) \times 25$

$= 550 + 225$

Production in 10th year was 775.

(iii)   For finding total production in first 7 years let's use formula for S_n.

$$S_n = \frac{n}{2}[2a+(n-1)d]$$
$$S_n = \frac{7}{2}[1100 + 150] = \frac{7}{2}[1250] = 7 \times 625 = 4375$$

Total production in first 7 years is 4375 mixers.

Ex. (2) Ajay sharma repays the borrowed amount of ₹ 3,25,000 by paying ₹ 30500 in the first month and then decreases the payment by ₹ 1500 every month. How long will it take to clear his amount?

Solution : Let the time required to clear the amount be n months. The monthly payment decreases by ₹ 1500. Therefore the payments are in A.P.

First term = a = 30500, d = -1500

Amount = S_n = 3,25,000

$$S_n = \frac{n}{2}[2a+(n-1)d]$$
$$3,25,000 = \frac{n}{2}[2 \times 30500+(n-1)d]$$
$$= \frac{n}{2}[2 \times 30500 - 1500n + 1500]$$
$$3,25,000 = 30500n - 750n^2 + 750n$$
$$\therefore 750n^2 - 31250n + 325000 = 0$$

divide both sides by 250.

$$\therefore 3n^2 - 125n + 1300 = 0$$
$$\therefore 3n^2 - 60n - 65n + 1300 = 0$$
$$\therefore 3n(n-20) - 65(n-20) = 0$$
$$\therefore (n - 20)(3n - 65) = 0$$
$$\therefore n - 20 = 0, \ 3n - 65 = 0$$
$$\therefore n = 20 \ \text{or} \ n = \frac{65}{3} = 21\frac{2}{3}$$

In an A.P. n is a natural number.

$$\therefore \ n \neq \frac{65}{3} \qquad \therefore \ n = 20$$

(Or, after 20 months, S_{20} = 3,25,000 then the total amount will be repaid. It is not required to think about further period of time.)

∴ To clear the amount 20 months are needed.

Ex. (3) Anvar saves some amount every month. In first three months he saves ₹ 200, ₹ 250 and ₹ 300 respectively. In which month will he save ₹ 1000?

Solution: Saving in first month ₹ 200; Saving in second month ₹ 250; .....

200, 250, 300, . . . this is an A.P.

Here $a = 200$, $d = 50$, Let's find n using t_n formula and then find S_n.

$$t_n = a+(n-1)d$$
$$= 200 +(n-1)50$$
$$= 200 + 50n - 50$$
$$1000 = 150 + 50n$$
$$150 + 50n = 1000$$
$$50n = 1000 - 150$$
$$50n = 850$$
$$\therefore n = 17$$

In the 17th month he will save ₹ 1000.

Let's find that in 17 months how much total amount is saved.

$$S_n = \frac{n}{2} [2a+(n-1)d]$$
$$= \frac{17}{2} [2 \times 200+(17-1) \times 50]$$
$$= \frac{17}{2} [400 + 800]$$
$$= \frac{17}{2} [1200]$$
$$= 17 \times 600$$
$$= 10200$$

In 17 months total saving is ₹ 10200.

Ex. (4) As shown in the figure, take point A on the line and draw a half circle P_1 of radius 0.5 with A as centre. It intersects given line in point B. Now taking B as centre draw a half circle P_2 of radius 1 cm which is on the other side of the line.
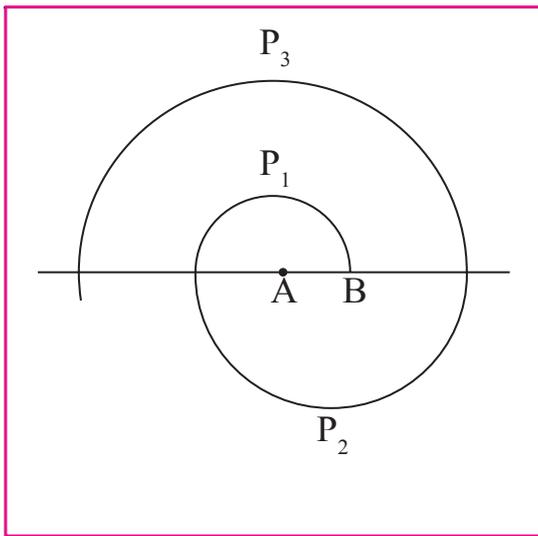
Now again taking A as centre draw a half circle P_3 of radius 1.5 cm. If we draw half circles like this having radius 0.5 cm, 1 cm, 1.5 cm, 2 cm, we get a figure of spiral shape. Find the length of such spiral shaped figure formed by 13 such half circles. ($\pi = \dfrac{22}{7}$)

Solution : Semi circumferences P_1, P_2, P_3, . . . are drawn by taking centres A, B, A, B,... It is given that radius of the first circle is 0.5 cm. The radius of the second circle is 1.0 cm,... From this information we will find P_1, P_2, P_3, . . . P_3.

Length of the first semi circumference = $P_1 = \pi\, r_1 = \pi \times \dfrac{1}{2} = \dfrac{\pi}{2}$

$P_2 = \pi\, r_2 = \pi \times 1 = \pi$

$P_3 = \pi\, r_3 = \pi \times 1.5 = \dfrac{3}{2}\pi$

The lengths are P_1, P_2, P_3, . . ., and the numbers $\dfrac{1}{2}\pi$, $1\,\pi$, $\dfrac{3}{2}\pi$, . . . are in A.P.

Here $a = \dfrac{1}{2}\pi$, $d = \dfrac{1}{2}\pi$, From this let's find S_{13}.

$$S_n = \dfrac{n}{2}\,[2a+(n-1)d]$$

$$S_{13} = \dfrac{13}{2}\,[2 \times \dfrac{\pi}{2} +(13-1) \times \dfrac{1}{2}\,\pi]$$

$$= \dfrac{13}{2}\,[\pi + 6\,\pi]$$

$$= \dfrac{13}{2} \times 7\,\pi =$$

$$= \dfrac{13}{2} \times 7 \times \dfrac{22}{7}$$

$$= 143 \text{ cm}.$$

∴ The total length of spiral shape formed by 13 semicircles is 143 cm.

Ex. (5) In the year 2010 in the village there were 4000 people who were literate. Every year the number of literate people increases by 400. How many people will be literate in the year 2020?

Solution :

Year	2010	2011	2012	$\cdots$	2020
Literate People	4000	4400	4800	$\cdots$	

$a = 4000, \quad d = 400 \quad n = 11$

$$t_n = a+(n-1)d$$

$$= 4000 + (11-1)400$$

$$= 4000 + 4000$$

$$= 8000$$

In year 2020, 8000 people will be literate.

Ex. (6) In year 2015, Mrs. Shaikh got a job with salary ₹ 1,80,000 per year. Her employer agreed to give ₹ 10,000 per year as increment. Then in how many years will her annual salary be ₹ 2,50,000?

Solution :

Year	First Year (2015)	Second Year (2016)	Third Year (2017)	$\cdots$
Salary (₹)	[1,80,000]	[1,80,000 + 10,000]		$\cdots$

$a = 1,80,000 \quad d = 10,000 \quad n = ? \quad t_n = 2,50,000$ ₹

$t_n = a+(n-1)d$

$2,50,000 = 1,80,000 + (n-1) \times 10,000$

$(n-1) \times 10000 = 70,000$

$(n-1) = 7$

$n = 8$

In the 8th year her annual salary will be ₹ 2,50,000.

1. On 1st Jan 2016, Sanika decides to save ₹ 10, ₹ 11 on second day, ₹ 12 on third day. If she decides to save like this, then on 31st Dec 2016 what would be her total saving?

2. A man borrows ₹ 8000 and agrees to repay with a total interest of ₹ 1360 in 12 monthly instalments. Each instalment being less than the preceding one by ₹ 40. Find the amount of the first and last instalment.

3. Sachin invested in a national saving certificate scheme. In the first year he invested ₹ 5000, in the second year ₹ 7000, in the third year ₹ 9000 and so on. Find the total amount that he invested in 12 years.

4. There is an auditorium with 27 rows of seats. There are 20 seats in the first row, 22 seats in the second row, 24 seats in the third row and so on. Find the number of seats in the 15th row and also find how many total seats are there in the auditorium?

5. Kargil's temperature was recorded in a week from Monday to Saturday. All readings were in A.P.The sum of temperatures of Monday and Saturday was 5° C more than sum of temperatures of Tuesday and Saturday. If temperature of Wednesday was −30° celsius then find the temperature on the other five days.

6. On the world environment day tree plantation programme was arranged on a land which is triangular in shape. Trees are planted such that in the first row there is one tree, in the second row there are two trees, in the third row three trees and so on. Find the total number of trees in the 25 rows.

Problem Set - 3

1. Choose the correct alternative answer for each of the following sub questions.

   (1) The sequence −10, −6, −2, 2, . . .

   (A) is an A.P., Reason $d = -16$ (B) is an A.P., Reason $d = 4$

   (C) is an A.P., Reason $d = -4$ (D) is not an A.P.

   (2) First four terms of an A.P. are ....., whose first term is −2 and common difference is −2.

   (A) −2, 0, 2, 4   (B) −2, 4, −8, 16

   (C) −2, −4, −6, −8   (D) −2, −4, −8, −16

   (3) What is the sum of the first 30 natural numbers?

   (A) 464   (B) 465   (C) 462   (D) 461

(4) For an given A.P. $t_7 = 4$, $d = -4$ then $a = \ldots$

    (A) 6   (B) 7   (C) 20   (D) 28

(5) For an given A.P. $a = 3.5$, $d = 0$, $n = 101$, then $t_n = \ldots$

    (A) 0   (B) 3.5   (C) 103.5   (D) 104.5

(6) In an A.P. first two terms are -3, 4 then 21st term is $\ldots$

    (A) -143 (B) 143   (C) 137   (D) 17

(7) If for any A.P. $d = 5$ then $t_{18} - t_{13} = \ldots$

    (A) 5   (B) 20   (C) 25   (D) 30

(8) Sum of first five multuiples of 3 is. . .

    (A) 45   (B) 55   (C) 15   (D) 75

(9) 15, 10, 5, . . . In this A.P. sum of first 10 terms is . . .

    (A) -75 (B) -125 (C) 75   (D) 125

(10) In an A.P. 1st term is 1 and the last term is 20. The sum of all terms is $= 399$ then $n = \ldots$

    (A) 42   (B) 38   (C) 21   (D) 19

2.   Find the fourth term from the end in an A.P. -11, -8, -5, . . . , 49.

3.   In an A.P. the 10th term is 46, sum of the 5th and 7th term is 52. Find the A.P.

4.   The A.P. in which 4th term is -15 and 9th term is -30. Find the sum of the first 10 numbers.

5.   Two A.P.'s are given 9, 7, 5, . . . and 24, 21, 18, ... If n^{th} term of both the progressions are equal then find the value of n and n^{th} term.

6.   If sum of 3rd and 8th terms of an A.P. is 7 and sum of 7th and 14th terms is -3 then find the 10th term.

7.   In an A.P. the first term is -5 and last term is 45. If sum of all numbers in the A.P. is 120, then how many terms are there? What is the common difference?

8.   Sum of 1 to n natural numbers is 36, then find the value of n.

9. Divide 207 in three parts, such that all parts are in A.P. and product of two smaller parts will be 4623.

10. There are 37 terms in an A.P., the sum of three terms placed exactly at the middle is 225 and the sum of last three terms is 429. Write the A.P.

11★ If first term of an A.P. is a, second term is b and last term is c, then show that sum of all terms is $\dfrac{(a+c)(b+c-2a)}{2(b-a)}$.

12.★ If the sum of first p terms of an A.P. is equal to the sum of first q terms then show that the sum of its first (p + q) terms is zero. (p ≠ q)

13.★ If m times the mth term of an A.P. is eqaul to n times n^{th} term then show that the $(m + n)^{th}$ term of the A.P. is zero.

14. ₹ 1000 is invested at 10 percent simple interest. Check at the end of every year if the total interest amount is in A.P. If this is an A.P. then find interest amount after 20 years. For this complete the following activity.

$$\text{Simple interest} = \dfrac{P \times R \times N}{100}$$

$$\text{Simple interest after 1 year} = \dfrac{1000 \times 10 \times 1}{100} = \boxed{}$$

$$\text{Simple interest after 2 year} = \dfrac{1000 \times 10 \times 2}{100} = \boxed{}$$

$$\text{Simple interest after 3 year} = \dfrac{\boxed{} \times \boxed{} \times \boxed{}}{100} = 300$$

According to this the simple interest for 4, 5, 6 years will be 400, $\boxed{}$, $\boxed{}$ respectively.

From this $d = \boxed{}$, and $a = \boxed{}$

Amount of simple interest after 20 years

$$t_n = a + (n-1)d$$
$$t_{20} = \boxed{} + (20-1)\boxed{}$$
$$t_{20} = \boxed{}$$

Amount of simple interest after 20 years is = $\boxed{}$

N5JRFH

4 ▸ Financial Planning

- GST - Introduction
- GST - Computation and ITC
- GST - Tax Invoice
- Shares, Mutual Funds and SIP

Let's discuss.

Teacher : Dear students, in our country which tax system is in practice for business ?

Ayush : GST system is in practice.

Teacher : Very good ! What do you know about GST ?

Ayan : **GST** stands for **G**oods and **S**ervice **T**ax.

Aisha : Yes, the whole country follows the same tax levy system.

Teacher : Correct, before GST every state had variety of taxes levied at different stages of trading. Observe the picture given below and tell which taxes existed before GST and are now subsumed in GST ?

Shafik : Taxes that existed before were Excise Duty, Custom Duty, VAT, Entertainment tax, Central sales tax, Service tax, Octroi etc.

Teacher : All these taxes are subsumed under GST, that is why GST is One nation, One tax, One market. GST is in effect from 1st of July 2017.

Excise Duty

Service Tax

Custom Duty

Central Surcharge and Cess

VAT

Luxury Tax

Entertainment Tax

Octroi

CST

Countervailing Duty (CVD)

Spectial Additional Duty (SAD)

Goods and Service Tax (GST)
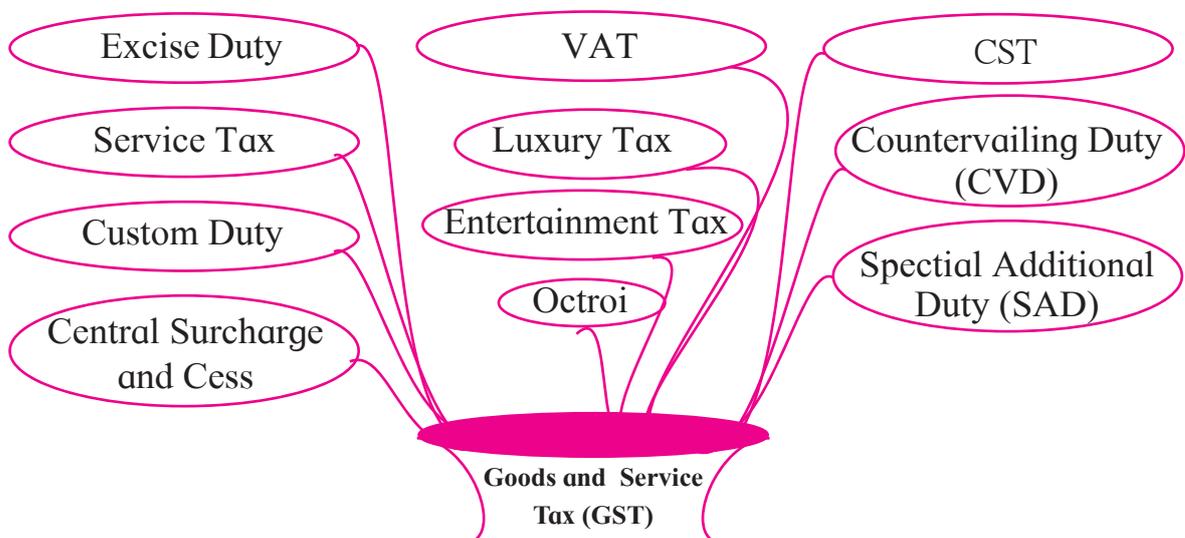
Tax Invoice

Tax Invoice of goods purchase (Sample)										
SUPPLIER : A to Z SWEET MART GSTIN : 27ABCDE1234H1Z5										
143, Shivaji Rasta, Mumbai : 400001, Maharashtra.										
Mo No. 92636 92111 e-mail - atoz@gmail.com										
Invoice No. GST/110 **Invoice Date**: 31-Jul-2017										
S. No.	HSN code	Name of Product	Rate	Quantity	Taxable Amount	CGST Rate	Tax	SGST Rate	Tax	Total ₹
1	210690	Pedhe	₹ 400 per kg.	500 gm.	200.00	2.5%	5.00	2.5%	5.00	210.00
2	210691	Chocolate	₹ 80	1Bar	80.00	14%	11.20	14%	11.20	102.40
3	2105	Ice-cream	₹ 200 per pack	1 pack (500 gm.)	200.00	9%	18.00	9%	18.00	236.00
4	1905	Bread	₹ 35	1 pack	35.00	0%	0.00	0%	0.00	35.00
5	210690	Butter	₹ 500 per kg.	250 gm	125.00	6%	7.50	6%	7.50	140.00
					Total Rupees		41.70		41.70	723.40

Ved : In the invoice we see some new words, please explain them.

Teacher : CGST and SGST are two components of GST. CGST is **C**entral **G**oods and **S**ervice **T**ax which is to be paid to the central government. Whereas SGST is **S**tate **G**oods and **S**ervice **T**ax which is to be paid to the state government.

Ria : What is in the right most corner with a long queue of numbers and alphabets?

Teacher : It is GSTIN, dealer's indentification number. (**GSTIN**- **G**oods and **S**ervice **T**ax **I**dentification **N**umber). GSTIN is mandatory for the dealer whose annual turn over in previous financial year exceeds rupees 20 lacs. You know that PAN has 10 alpha-numerals, similarly GSTIN has 15 alpha-numerals. It includes 10 digit PAN of the dealer.

e.g. : 27 A B C D E 1 2 3 4 H 1 Z 5

10 digit PAN of the firm

1 represents one registration

Uniform for all (By default)

Check sum digit (confirms the validity of GSTIN when entered on GST portal)

2 digits for state code

Note : Here 27 is the state code of Maharashtra. From 27, one can understand that a person or a firm is registered in Maharashtra.

Jennie : There is a word HSN code in the tax invoice.

Teacher : All Goods are classified by giving numerical code called HSN code. It is to be quoted in the tax invoice. Full form of **HSN** is **H**armonized **S**ystem of **N**omenclature.

Joseph : As usual there is name of the shop, address, state, date, invoice number, mobile number and e-mail ID also in the tax invoice.

Teacher : Now we will see how the GST is charged for each product (Goods) in the bill. Observe the given bill and fill in the boxes with the appropriate number. Price of 1 kg of Pedhe is ₹ 400, therefore cost of 500 gm. of Pedhe is ₹ 200.

✦ CGST at the rate of 2.5% is ₹ [   ] and SGST at the rate of [   ] % is ₹ 5.00.

✦ It means that the rate of GST on Pedhe is 2.5+2.5=5% and hence the total GST is ₹ 10.

✦ The rate of GST on chocolate is [   ] % and hence the total GST is ₹ [   ]

✦ Rate of GST on Ice-cream is [   ] %, hence the total cost of ice-cream is ₹ [   ]

✦ On butter CGST rate is [   ] % and SGST rate is also [   ] %. So GST rate on butter is [   ] %.

Aditya : Rate of GST on bread is 0 %. The rate of CGST and SGST is same for each product.

Ninad : Rates of GST are different for diferent products such as 0%, 5%, 12%, 18% and 28%.

Teacher : These rates are fixed and prescribed by the government. Now let us observe the tax invoice of services provided. Fill in the blanks with the help of given information.

Tax invoice of services provided (Sample)								
Food Junction, Khed-Shivapur, Pune Invoice No. 58								
Mo. No. 7588580000 email - ahar.khed@yahoo.com								
GSTIN : 27 AAAAA5555B1ZA Invoice Date 25-Dec-2017								
SAC	Food items	Qty	Rate (in ₹)	Taxable amount	CGST		SGST	
9961	Coffee	1	20	20.00	2.5%	. . .	2.5%	. . .
9963	Masala Tea	1	10	10.00	. . . .	. . .	2.5%	. . .
9962	Masala Dosa	2	60	. . . .	2.5%	. . .	. . .	. . .
			Total	. . .		. . .	. . .	. . .
Grand Total = ₹ --------								

Teacher : Compare both, Goods and Service Tax invoices and find the difference in codes.

Patrick : In the tax invoice for Goods, there is HSN code while in service invoice there is SAC.

Teacher : Services are also classified and special code numbers are given. These are called SAC or **S**ervice **A**ccounting **C**ode.

Sr. No.	Types	Rate of GST	Goods and services items list
I	Zero rated	0%	**Goods** - Essential Commodities like food grains, fruits, vegetables, milk, salt, earthen pots etc. **Services** - Charitable trust activities, transport of water, use of roads and bridges, public library, agriculture related services, Education and Health care services etc.
II	Low rated	5%	**Goods**- Commonly used items- LPG cylinder, Tea, coffee, oil, Honey, Frozen vegetables, spices, sweets etc. **Services** - Railway transport services, bus transport services, taxi services, Air transport (economy class), Hotels providing food and beverages etc.
III	Standard rated (I slab)	12%	**Goods**- Consumer goods : Butter, Ghee, Dry fruits, Jam, Jelly, Sauces, Pickles. Mobile phone etc. **Services** - Printing job work, Guest house, Services related to construction business.
IV	Standard rated (II slab)	18% (Most of the Goods and services are included)	**Goods** - Marble, Granite, Perfumes, Metal items, Computer, Printer, Monitor, CCTV etc. **Services** - Courier services, Outdoor catering, Circus, Drama, Cinema, Exhibitions, Currency exchange, Broker Services in share trading etc.
V	Highly rated	28%	**Goods** - Luxury items, Motor Cycles and spare parts, Luxury cars, Pan-masala, Vacuum cleaner, Dish washer, AC, Washing machine, Fridge, Tobacco products, Aerated water etc. **Services** - Five star Hotel accommodation Amusement parks, Water parks, Theme parks, Casino, Race course, IPL games, Air transport (business class) etc.

Reference : www.cbec.gov.in (Central Board of Excise & Customs)

Besides these rates, find on which goods are the GST rates levied between 0 and 5 ?

Note : – The rates and types of GST are as prescribed by the government at the time of writing this chapter. GST rates are subject to change. Electricity, petrol, diesel etc are not under purview of GST.

Activity I : Make a list of ten things you need in your daily life. Find the GST rates with the help of GST rate chart given here, News papers or books, internet, or the bills of purchases. Verify these rates with the list prepared by your friends.

Goods	Rate of GST	Goods	Rate of GST
1. Sketch book	- - - - -	6. - - - --	- - - - -
2. Compass-box	- - - - -	7.- - - - -	- - - - -
3. - - - - -	- - - - -	8. - - - - -	- - - - -
4. - - - - -	- - - - -	9. - - - - -	- - - - -
5. - - - - -	- - - - -	10. - - - - -	- - - - -

Activity II : Make a list of ten services and their GST rates as per actvity I. (e.g. Railway and ST bus booking services etc.) You can also collect service bills and complete the given information

Services	Rate of GST	Services	Rate of GST
1.Railway booking	- - - - -	6. - - - --	- - - - -
2.Courier Services	- - - - -	7.- - - - -	- - - - -
3.- - - --	- - - - -	8. - - - - -	- - - - -
4.- - - - -	- - - - -	9. - - - - -	- - - - -
5.- - - - -	- - - - -	10. - - - - -	- - - - -

Activity III : Complete the given table by writing remaining SAC and HSN codes with rates and add some more items in the list.

Services	SAC	GST rate	Goods	HSN	GST rate
Railway transport services	996511	--	Dulux paint	3208	28%
Airways services (economy)	996411	--	Ball bearing	84821011	28%
Currency exchange services	997157	--	Speedometer	8714	28%
Brokerage services	997152	--	Potatoes	0701	0%
Taxi services	996423	--	--	--	--
Five-star Hotel services	9963	--	--	--	--
--	--	--	--	--	--

Activity IV : Prepare a chart of Goods and Services as in activity III with codes and GST rates. Stick or draw the pictures of Goods and services to enhance your activity.

Note : Rates on goods and services, SAC and HSN codes are only for information, no need to remember them.

Project : Collect various Goods and Service tax invoices. Study these invoices with reference to GST and discuss with your classmates.

Ex. (1) Arati Gas Agency supplied LPG cylinder to the consumer for taxable value of ₹ 545. GST charged is 5%. What is the amount of CGST and SGST in the tax invoice ? What is the total amount paid by the consumer ? Find the amount of GST to be paid by Arati Gas Agency.

Solution : Rate of GST = 5% ∴ Rate of CGST 2.5%, and Rate of SGST = 2.5%.

$$CGST = \frac{2.5}{100} \times 545 = 13.625 = ₹\ 13.63$$

∴ SGST = CGST = ₹ 13.63

　　　Amount paid by the consumer = Taxable value + CGST + SGST

　　　　　　　　　　　　　　　　= 　　545 + 13.63 + 13.63

　　　　　　　　　　　　　　　　= 　　572.26

Arati Gas Agency has to pay CGST = ₹ 13.63. and SGST = ₹ 13.63

∴ Total GST to be paid = 13.63 × 2 = ₹ 27.26.

Ex. (2) Courier service agent charged total ₹ 590 to courier a parcel from Nashik to Nagpur. In the tax invoice taxable value is ₹ 500 on which CGST is ₹ 45 and SGST is ₹ 45. Find the rate of GST charged for this service.

Solution : Total GST = CGST + SGST = 45 + 45 = ₹ 90.

$$Rate\ of\ GST = \frac{90}{500} \times 100 = 18\%$$

∴ Rate of GST charged by agent is 18%.

Ex. (3) Shreekar bought a Laptop with 10% discount on printed price. The printed price of that Laptop was ₹ 50,000. 18% GST was charged on discounted price. Find the amount of CGST and SGST. What amount did Shreekar pay ?

Solution : Discount = 10% of 50,000 = ₹ 5,000

∴ Taxable value of Laptop = 50,000 - 5,000 = ₹ 45,000.

∴ Rate of GST = 18% (Given)　∴ Rate of CGST = 9%

∴ CGST = 　9% of 45,000 = $\frac{9}{100} \times 45000$ = ₹ 4050.

∴ SGST = ₹ 4050.

∴ Amount paid = 45000 + 4050 + 4050 = ₹ 53,100.

Ans. Shreekar paid ₹ 53,100 for the Laptop.

Note : Value of Goods on which GST is levied is called taxable value. Total value or Invoice value is the value with GST. If not mentioned take the selling prices as taxable price. Remember that in tax invoice CGST amount is always equal to SGST amount.

Practice Set 4.1

1. 'Pawan Medical' supplies medicines. On some medicines the rate of GST is 12%, then what is the rate of CGST and SGST?

2. On certain article if rate of CGST is 9% then what is the rate of SGST? and what is the rate of GST?

3. 'M/s. Real Paint' sold 2 tins of lustre paint and taxable value of each tin is ₹ 2800. If the rate of GST is 28%, then find the amount of CGST and SGST charged in the tax invoice.

4. The taxable value of a wrist watch belt is ₹ 586. Rate of GST is 18%. Then what is price of the belt for the customer ?

5. The total value (with GST) of a remote-controlled toy car is ₹ 1770. Rate of GST is 18% on toys. Find the taxable value, CGST and SGST for this toy-car.

6. 'Tiptop Electronics' supplied an AC of 1.5 ton to a company. Cost of the AC supplied is ₹ 51,200 (with GST). Rate of CGST on AC is 14%. Then find the following amounts as shown in the tax invoice of Tiptop Electronics.

   (1) Rate of SGST   (2) Rate of GST on AC (3) Taxable value of AC

   (4) Total amount of GST (5) Amount of CGST (6) Amount of SGST

7. Prasad purchased a washing-machine from 'Maharashtra Electronic Goods'. The discount of 5% was given on the printed price of ₹ 40,000. Rate of GST charged was 28%. Find the purchase price of washing machine. Also find the amount of CGST and SGST shown in the tax invoice.

Let's learn.

GST in trading chain

Manufacturer → Wholesaler or Distributor → Retailer → Consumer
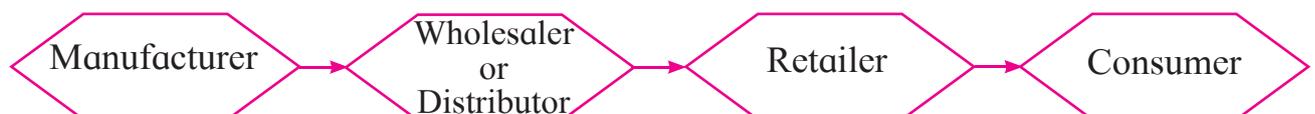
Trading Chain (within state)

Let's learn through an example how GST is charged and paid to the government at every stage of trading.

Illustration : Suppose manufacturer of a watch has sold one watch for

₹ 200. (including profit) to the wholsaler. Wholesaler sold that watch for ₹ 300 to the retailer. Retailer sold it to the customer for ₹ 400. Rate of GST charged at every stage is 12%. Then how each trader pays GST and takes his input tax credit (ITC) at every stage of transaction is shown in the following flow-chart. Observe and study it.

Explanation :

Here three financial transactions took place till the watch from manufacturer reaches to the customer. How the taxes are charged, collected and paid to the central government and state government at each stage is shown below. The statement of taxes paid is given in the table thereafter.
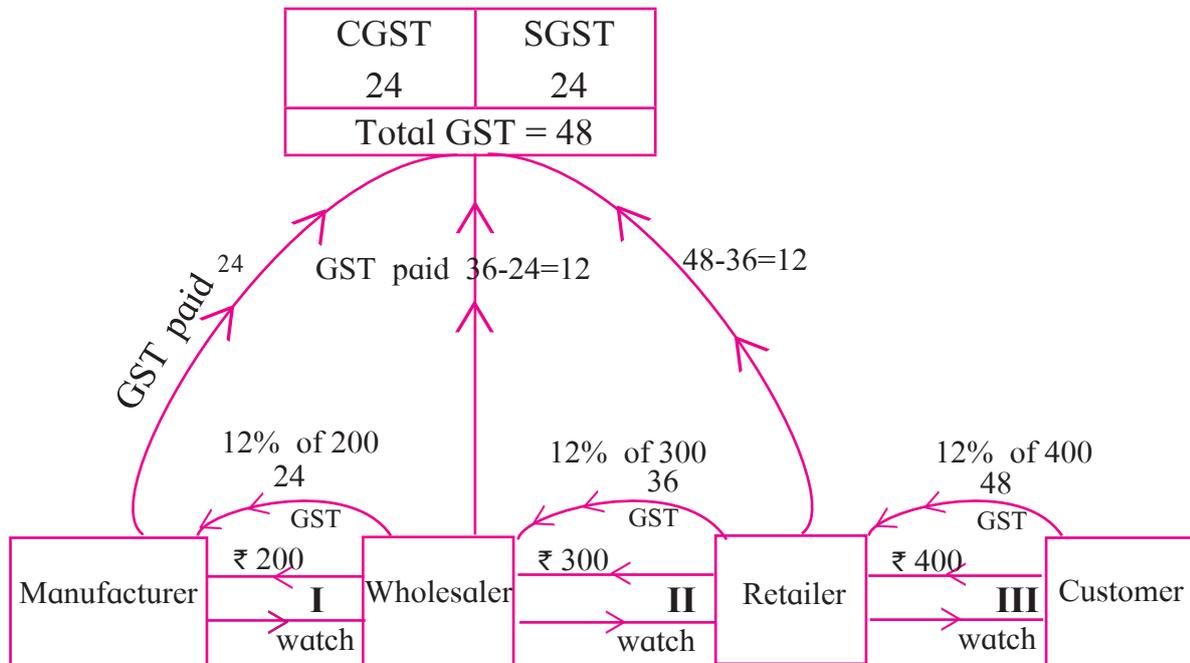
Here all the three financial transactions took place in one state. Therefore three tax invoices were generated as follows. Each tax invoice shows the brief computation of GST.

GST in Tax Invoice I		
Price of watch = ₹ 200		
CGST 6%	=	₹ 12
SGST 6%	=	₹ 12
Total	=	₹ 224

Tax invoice of manufacturer
B2B

GST in Tax Invoice II		
Price of watch = ₹ 300		
CGST 6%	=	₹ 18
SGST 6%	=	₹ 18
Total	=	₹ 336

Tax invoice of wholesaler
B2B

GST in Tax Invoice III		
Price of watch = ₹ 400		
CGST 6%	=	₹ 24
SGST 6%	=	₹ 24
Total	=	₹ 448

Tax invoice of relailer
B2C

- Trading between GSTIN holders is known as Business to Business, in short B2B. Trading between GSTIN holder and consumer is known as Business to Consumer, in short B2C. This is the last link in the trading chain.

Bifurcation of taxes paid to the government by the traders at each stage.

	CGST		SGST		Total GST paid
• By the manufacturer	₹ 12	+	₹ 12	=	₹ 24
• By the wholesaler	₹ 6	+	₹ 6	=	₹ 12
• By the retailer	₹ 6	+	₹ 6	=	₹ 12
	₹ 24	+	₹ 24	=	₹ 48

Note : Observe that at every stage, a trader has paid GST after subtracting the tax he paid at the time of purchase from the tax he collected at the time of sale.

At the end the customer paid ₹ 448 for the watch. So the total tax paid by the traders was indirectly paid by the customer. So GST is a type of indirect tax. In this case, the wholesaler and retailer used their input tax as credit and got back all the GST paid by them.

What is Input Tax Credit ? (ITC)

GST is levied and collected at every stage of trading from manufacturer to consumer. When trader pays GST at the time of purchase, it is called **'Input tax'** and he collects GST at the time of sale which is called **'Output tax'.** At the time of paying GST to the government a trader deducts the input tax from the output tax and pays the remaining tax. This deduction of input tax is called **Input Tax Credit**.

GST payable = Output tax – ITC

In short, while paying taxes to the government each trader in the trading chain subtracts the tax paid at the time of purchase from the tax collected at the time of sale and pays the remaining tax.

Ex. (1) Mr. Rohit is a retailer. He paid GST of ₹ 6500 at the time of purchase. He collected GST of ₹ 8000 at the time of sale. (i) Find his input tax and out put tax. (ii) What is his Input tax credit ? (iii) Find his payable GST. (iv) Hence find the payable CGST and payable SGST.

Solution : Mr. Rohit's payable GST means, GST to be paid to the government by Mr. Rohit.

(i) Output tax (tax collected at the time of sale) = ₹ 8000.

(ii) Input tax (tax paid at the time of purchase) = ₹ 6500

∴ ITC = ₹ 6500.

(iii) GST payable = Output tax - ITC

= ₹ 8000 - ₹ 6500 = ₹1500

(iv) ∴ payable CGST = $\dfrac{1500}{2}$ = ₹ 750 and payable SGST = ₹ 750.

Ex. (2) M/s. Jay Chemicals purchased a liquid soap having taxable value ₹ 8000 and sold it to the consumers for the taxable value ₹ 10,000. Rate of GST is 18%. Find the CGST and SGST payable by M/s. Jay Chemicals.

Solution :

Input Tax = 18% of 8000

$= \dfrac{18}{100} \times 8000 = $ ₹ 1440.

Output Tax = 18% of 10,000

$= \dfrac{18}{100} \times 10000 = $ ₹ 1800

∴ GST payable = Output tax - ITC

= 1800 - 1440

= ₹ 360

∴ payable CGST = ₹ 180 and payable SGST = ₹ 180 by M/s. Jay Chemicals

Ex. (3) M/s. Jay Chemicals purchased a liquid soap for ₹ 8000 (with GST) and sold it to the consumers for ₹ 10,000 (with GST). Rate of GST is 18%. Find the amount of CGST and SGST to be paid by Jay Chemicals

Solution : Note that here the prices are including GST.

Total value (value with GST) = Taxable value + GST

If the taxable value of liquid soap is ₹ 100, then the total value is ₹ 118.

The ratio of $\dfrac{\text{Total value}}{\text{Taxable Value}}$ is constant as the rate of GST is same.

i) For total value of ₹ 118, the taxable value is ₹ 100 and for total value of ₹ 8000, let the taxable value be ₹ x .

$$\therefore \frac{x}{8000} = \frac{100}{118}$$

$$\therefore x = \frac{8000}{118} \times 100 = ₹ \ 6779.66$$

$\therefore$ GST paid at the time of purchase = 8000 - 6779.66

Input tax = ₹ 1220.34 $\therefore$ ITC = ₹ 1220.34 . . . . . (I)

ii) For total value of ₹ 10,000 let the taxable value be ₹ y.

$$\therefore \frac{y}{10000} = \frac{100}{118}$$

$$\therefore y = \frac{10,00,000}{118} = ₹ \ 8474.58 .$$

$\therefore$ Output tax (tax collected) = 10000.00 - 8474.58

$$= ₹ 1525.42. . . . . (II)$$

$\therefore$ GST payable = Output tax - Input tax = 1525.42 - 1220.34

$$= ₹ \ 305.08.$$

$\therefore$ payable CGST = payable SGST = 305.08 $\div$ 2 = ₹152.54

Ans. : Jay Chemicals has to pay ₹ 152.54 CGST and ₹152.54 SGST.

Note : Observe Ex. 2 and Ex. 3 carefully. Both the types of 'Tax Invoices' are commonly used. While purchasing goods, ask the shopkeeper whether the printed price includes GST.

ICT Tools or Links.

Ex. (4) Suppose a manufacturer sold a cycle for a taxable value of ₹ 4000 to the wholesaler. Wholesaler sold it to the retailer for ₹ 4800 (taxable value). Retailer sold it to a customer for ₹ 5200 (taxable value). Rate of GST is 12%. Complete the following activity to find the payable CGST and SGST at each stage of trading.

Solution : Trading chain
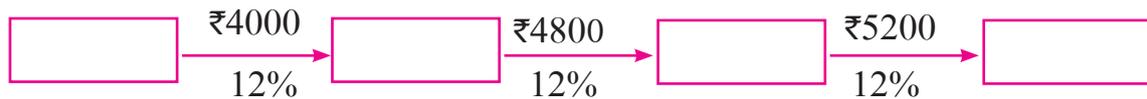
Output tax of manufacturer = 12% of 4000 = $\boxed{\ldots \times \dfrac{\cdots}{\cdots}}$ = $\boxed{}$

GST payable by manufacturer = ₹480

Output tax of wholesaler = 12% of 4800 = ₹ 576

∴ GST payable by wholesaler = Output tax - Input tax

$$= \boxed{576} - \boxed{480}$$

$$= ₹\,96$$

Output tax of retailer = 12% of 5200 = $\boxed{}$

∴ GST payable by Retailer = Output tax of retailer - ITC of retailer

$$= \boxed{} - \boxed{}$$

$$= \boxed{}$$

Statement of GST payable at each stage of trading

Individual	GST payable	CGST payable	SGST payable
Manufacturer	₹ 480	₹ 240	₹
Wholesaler	₹ 96	₹	₹
Retailer	₹	₹	₹
Total	₹	₹	₹

Practice Set 4.2

1. 'Chetana Store' paid total GST of ₹ 1,00,500 at the time of purchase and collected GST ₹ 1,22,500 at the time of sale during 1ˢᵗ of July 2017 to 31ˢᵗ July 2017. Find the GST payable by Chetana Stores.

2. Nazama is a proprietor of a firm, registered under GST. She has paid GST of ₹ 12,500 on purchase and collected ₹ 14,750 on sale. What is the amount of ITC to be claimed ? What is the amount of GST payable ?

3. Amir Enterprise purchased chocolate sauce bottles and paid GST of ₹ 3800. He sold those bottles to Akbari Bros. and collected GST of ₹ 4100. Mayank Food Corner purchased these bottles from Akabari Bros and paid GST of ₹ 4500. Find the amount of GST payable at every stage of trading and hence find payable CGST and SGST.

4. Malik Gas Agency (Chandigarh Union Territory) purchased some gas cylinders for industrial use for ₹ 24,500, and sold them to the local customers for ₹ 26,500. Find the GST to be paid at the rate of 5% and hence the CGST and UTGST to be paid for this transaction. (for Union Territories there is UTGST instead of SGST.)

5. M/s Beauty Products paid 18% GST on cosmetics worth ₹ 6000 and sold to a customer for ₹ 10,000. What are the amounts of CGST and SGST shown in the tax invoice issued ?

6. Prepare Business to Consumer (B2C) tax invoice using given information. Write the name of the supplier, address, state, Date, invoice number, GSTIN etc. as per your choice. Supplier : M/s - - -- - Address- - - - - State - - - - - Date - - - - - - -

   Invoice No. - - - - - GSTIN - - - - - - - - - - - - - -

   Particulars - Rate of Mobile Battery - ₹ 200 Rate of GST 12%   HSN 8507, 1 pc.

   Rate of Headphone - ₹ 750   Rate of GST 18% HSN 8518, 1 pc.

(7) Prepare Business to Business (B2B) Tax Invoice as per the details given below. name of the supplier, address, Date etc. as per your choice.

Supplier - Name, Address, State, GSTIN, Invoice No., Date

Recipient - Name, Address, State, GSTIN,

Items : (1) Pencil boxes 100, HSN - 3924, Rate - ₹ 20, GST 12%

(2) Jigsaw Puzzles 50, HSN 9503, Rate - ₹ 100 GST 12%.

For More Information

Composition Scheme

The person whose annual turn over in the previous financial year is less than 1.5 crore can opt for composition scheme under GST rules. GST rates applicable to composition dealers are as follows.

GST rates for composition Scheme

Sr. No.	Supplier	GST rate	(CGST + SGST)
1.	Restaurants	5%	2.5% + 2.5%
2.	Manufacturers & traders	1%	0.5% + 0.5%

Some rules for composition dealers

• Composition dealers cannot collect tax from the customers, hence they can not issue tax invoice. They have to give '**bill of supply**'.

• Composition dealers should file the return quarterly (i.e. every 3 months.)

• Composition dealers cannot sell goods outside the state (Inter-state sale is not allowed) But they can purchase goods from other states.

• Composition dealers cannot avail the benefits of ITC.

• On the signboard of the shop, he should mention 'Composition taxable person'.

• On the Bill of supply it is mandatory to print 'Composition taxable person not eligible to collect tax on supplies' in bold letters.

Features of GST

- Many Indirect Taxes are subsumed **under GST.**

- No dispute between Goods and Services.

- Statewise Registration for traders.

- **GSTIN holder needs to keep all the records and should pay GST** in time.

- Transparency in transactions.

- This tax system is simple and easy to understand .

- Removal of cascading effect of taxes hence the prices are controlled

- Increase in Quality of Goods and Services as they are globally competetive.

- Boost to 'Make in India' project.

- Technology driven tax system leads to speedy decisions.

- Goods and Service Tax system is a Dual model, as equal amount of tax is levied by Central and State governments.

Types of taxes under GST
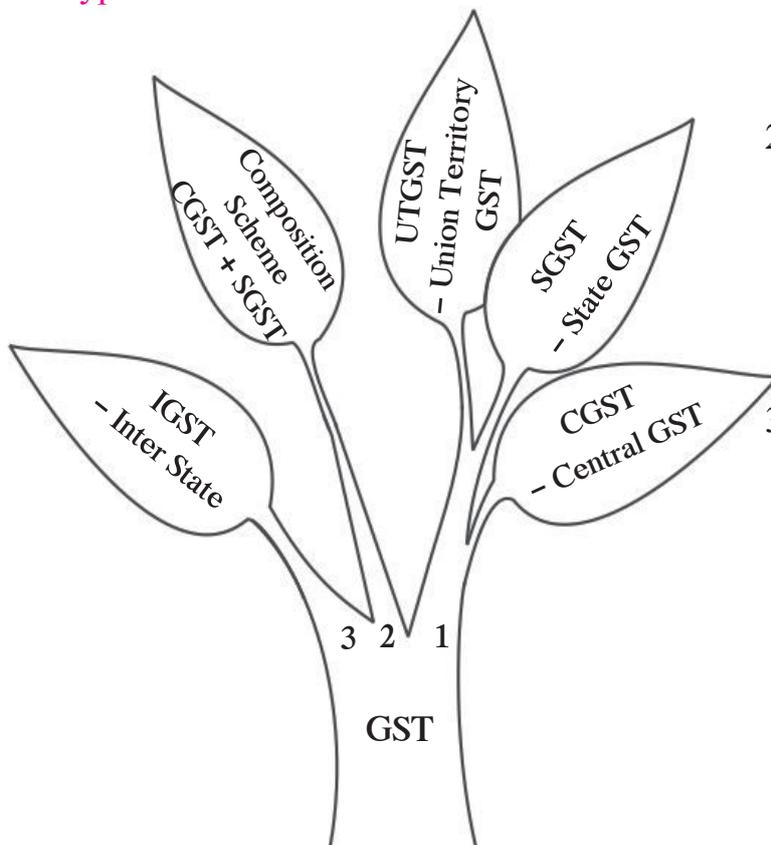
1. CGST-SGST (UTGST): Tax levied for trading within state (Intra state).

2. Composition Scheme : For those GSTIN holders whose annual turnover is between 20 lacs to 1.5 crore. They pay CGST and SGST with special rates.

3. IGST : Tax levied by central government for Inter state trading.

IGST- Integrated GST (for Inter state trade)

When trading of goods and services takes place between two or more states, the GST is levied only by the Central Government, and it is termed as IGST, hence the total amount is paid to the Central Government.

Suppose if a trader buys goods from another state and sells them in his state, then let us see how he can avail of the ITC, which he has paid as IGST at the time of purchase.

For example : Trader 'M' (of Maharashtra) purchased scooter parts for ₹ 20,000 from trader 'P' (of Punjab) and paid tax of ₹ 5600 as IGST (GST rate 28%) to the trader 'P'.

Trader 'M' sold these parts to local consumers for ₹ 25,000 and collected ₹ 7000 GST at the rate of 28%, bifurcated as CGST ₹ 3500 + SGST ₹ 3500

At the time of paying taxes to the Government, see, how to take ITC of ₹ 5600.

Note : For taking credit of IGST first preference to be given to pay the liability of IGST then CGST and remaining amount can be utilised to pay SGST. Here there is no IGST during the sale for trader 'M', so first the credit is used for CGST and then for SGST.

CGST payable = 3500 – 3500 = 0 ₹

So out of ₹ 5600 credit of ₹ 3500 is utilised for CGST and the remaining amount 5600 - 3500 = 2100 is the credit available for SGST

∴ **SGST payable = 3500 - 2100 = 1400 ₹**

Trader 'M' has to pay ₹ 1400 as SGST.

Note that, trader 'M' got full credit of ₹ 5600. (so that ITC is completely utilised)

- -

Rule for availing ITC

ITC paid at the time of purchase   Tax collected (output liabilities)

utilised for IGST

Firstly ₹ (0)

While taking credit of IGST   *Secondly* (₹ 3500)   utilised for CGST

(₹ 5600)   *At the end* (₹ 2100)   utilised for SGST

Hence payable SGST is ₹ 1400
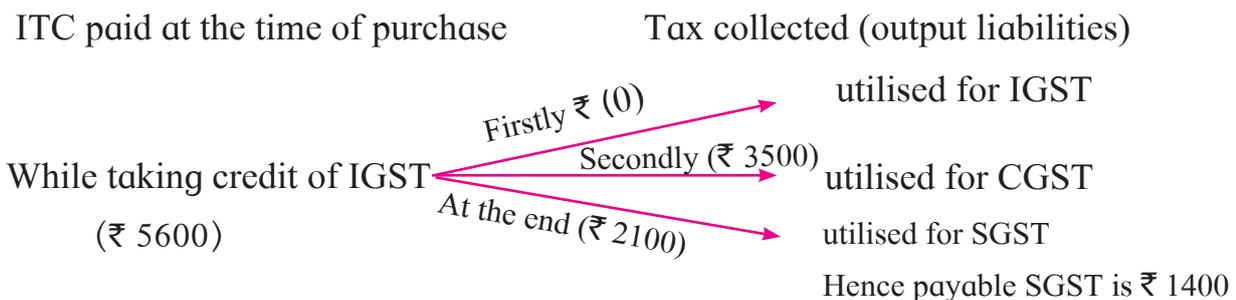
In the previous class we have learnt the importance of savings and investments, which you might have started practising whenever possible. We develop good habits to maintain physical health, in the same way we should develop a habit of saving and investing regularly to maintain financial health. There are many different ways of investments. So deep study and experience both are essential.

Let's discuss.

Shweta is working in a company. From this month her salary increased by 5% and in the next month she will also get bonus. She is thinking of investing this increment. Her friend Neha is working in the office of a financial advisor, so she can advise Shweta in this matter. Neha told, 'It is important to have diversification in one's investments. e.g. you should think of investing in life insurance, health insurance, owning a house, FD's and recurring account in the bank etc...' Shweta said, 'I have insurance and FD in the bank. Even Provident Fund is deducted regularly from my salary. What are the other ways?' Neha answered, ''Investing in shares, Mutual Funds (MF), Debentures, Bonds etc is more popular these days. Inclination of people towards SIP is also increasing. Well I think, you are getting salary increment every month, so **S**ystematic **I**nvestment **P**lan (SIP) is suitable for you.''
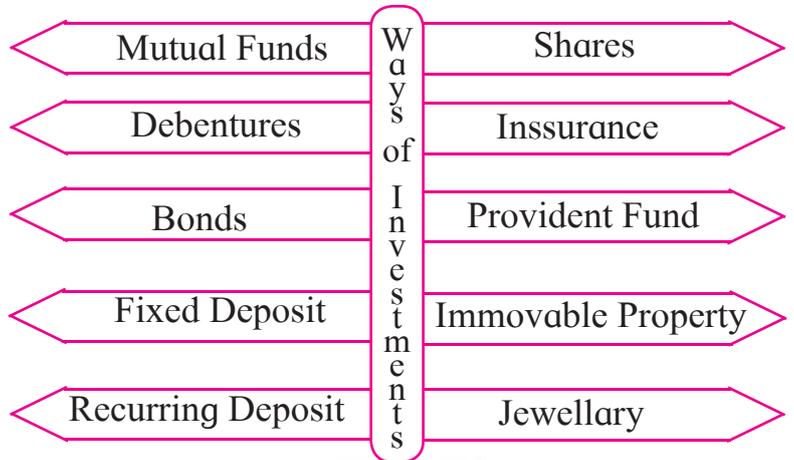
Ways of Investments:
Mutual Funds — Shares
Debentures — Insurance
Bonds — Provident Fund
Fixed Deposit — Immovable Property
Recurring Deposit — Jewellary

Piggy bank Savings

We hear such dialogues every now and then. So we all must have the current information beneficial for all as it says 'बहुजन हिताय, बहुजन सुखाय'.

In this chapter we shall learn about shares, mutual funds and SIP before actually investing in them.

Shares

To own a shop is proprietorship. When two or more individuals coming together to carry out a business is a partnership, which requires small capital. To establish a company, desiring persons come together and form a company. Company is to be registered under the Indian Companies' Act, 1956. Persons who form a company are called Promoters and the company is called Limited Company.

Amount required to start a company is called **Capital**. This capital is divided into small equal parts, each part is of ₹ 1, ₹ 2, ₹ 5, ₹ 10 or ₹100 etc. This small part is called share of the company. These shares are sold in the sharemarket to raise the capital.

Share : A share is the smallest unit of the capital. The value of a share is printed on the company's certificate with other details and it is called a share certificate.

Share Holder : A person who owns the share is called a share holder. The shareholder is a part owner of the company in the proportion of number of shares he/she holds.

Stock Exchange : It is a place where buying and selling of shares take place. It is also known as share market or stock market, equity market or capital market. Companies should be listed in the stock market for trading.

Face Value (FV) : The value printed on the share certificate is called the Face value of the share. It is also called Nominal value or Printed value or par value.

Market Value (MV) : The price at which the shares are sold or purchased in the stock market is called Market value (MV) of the share.

In the live sharemarket the Market Value changes frequently.

If the company's performance is better than expected, then those shares are in demand. The number of shares is fixed, therefore share supply could not be increased and hence the share price increases. If the company is not doing well, the share price falls. [Increase in price is shown by ▲ (green triangle upward), and decrease in price is shown by ▼ (Red triangle downward).] This is the reason for increase or decrease in SENSEX and NIFTY index.

Dividend : The part of annual profit of a company which is distributed per share among shareholders is called dividend. If the company is performing well then the value of share capital increases hence the price of the share goes up. As a result company gives good devidend. For the sharehlders the devidend income is taxfree.

Let's remember!

Whatever may be the market value, the dividend is always reckoned on the Face Value of a share.

For more Information :

There are two main stock exchanges, of India, **BSE (Bombay Stock Exchange**) and **NSE (National Stock Exchange).** BSE is the oldest in Asia while NSE is the India's largest stock Exchange.

There are two share indices namely - SENSEX and NIFTY which reflect the overall market sentiments. SENSEX is **SENS**itive + ind**EX**. Which was introduced by BSE on 1-1-1986. SENSEX is detemined from 30 stocks. They are the stocks of well established and financially sound companies from the main sectors.

NIFTY as the name suggests is made up of two words that is **N**SE and F**IFTY** which was introduced by NSE. It depends on India's topmost outperforming 50 companies.

ICT Tools or Links.

Visit the website of SEBI. Also get information of share market from TV channels, BSE, NSE websites or watch videos on internet. There you can see two strips continuously flashing advances and declines in the market value of shares. Generally the upper strip shows BSE shares while lower strip shows NSE shares. Also find out what is the book value of shares from the available resources.

Comparison of FV and MV

(1) If MV > FV then the share is at premium.

(2) If MV = FV then the share is at par.

(3) If MV < FV then the share is at discount.

For example : (1) suppose FV = ₹ 10, MV = ₹ 15 and 15 - 10 = ₹ 5

∴ The share is at premium of ₹ 5, as MV > FV

(2) suppose FV = ₹ 10, MV = ₹ 10 and 10 - 10 = 0

∴ The share is at par. As MV = FV

(3) suppose FV = ₹ 10, MV = ₹ 7 and 10 - 7 = 3

∴ The share is at discount. As MV < FV.

Sum Invested : Total amount required to purchase the shares is sum invested.

Sum invested = Number of shares × MV

Ex. (1) If 50 shares of FV ₹ 100 each are purchased for MV ₹ 120. Find the sum invested.

Solution : Sum invested = number of shares × MV

$$= 50 \times 120 = ₹\ 6000$$

Ex. (2) If you want to purchase 50 shares of MV ₹ 50 each. What is the total amount to be paid ?

Solution : Sum invested = Number of shares × MV = 50 × 50 = ₹ 2500

Rate of Return - RoR

When we invest in shares, it is important to know the return on investment. Observe the following example.

Ex. (1) Shriyash purchased a share of FV ₹ 100 for MV of ₹ 120. Company declared 15% dividend on the share. Find the rate of return.

Solution : FV = ₹ 100, MV = ₹ 120 D = Dividend = 15% per share.

Remember here, that on investment of ₹ 120 Shriyash got ₹ 15.

Let the rate of return be $x\%$

∴ $\dfrac{15}{120} = \dfrac{x}{100}$ ∴ $x = \dfrac{15 \times 100}{120} = \dfrac{25}{2} = 12.5\%$

> If 120 : 15
> then 100 : x

Ans. The rate of return for Shriyash is 12.5%.

Ex. (2) FV = ₹ 100, premium = ₹ 65 then MV = ?

Solution : MV = FV + Premium = 100 + 65 = ₹ 165.

Market value is ₹ 165 per share.

Ex. (3) Complete the following table using given information.

Sr.No.	FV	Share is at	MV
(i)	₹ 10	Premium of ₹ 7	
(ii)	₹ 25		₹ 16
(iii)		at par	₹ 5

Solution : (i) MV=10+7=₹17 (ii) at discount of 25−16=₹9 (iii) FV=₹5.

Ex. (4) Neel has invested in shares as follows. Find his total investment.

Company A : 350 shares, FV = ₹ 10, premium = ₹ 7

Company B : 2750 shares, FV = ₹ 5, Discount = ₹ 1.

Company C : 50 shares, FV = ₹ 100, MV = ₹ 150.

Solution : Company A : Premium = ₹ 7   MV = FV + Premium

= 10 + 7 = ₹ 17.

∴ Investment in company A = Number of shares × MV

= 350 × 17 = ₹ 5950.

Company B : FV = ₹ 5, MV = ₹ 4.

∴ Investment in company B = Number of shares × MV

= 2750 × 4 = ₹ 11,000.

Company C : FV = ₹ 100, MV = ₹ 150.

∴ Investment in company C = Number of shares × MV

= 50 × 150 = ₹ 7500.

Ans. Neel has invested 5950 + 11000 + 7500 = ₹ 24,450.

Ex. (5) Smita has invested ₹ 12,000 and purchased shares of FV ₹ 10 at a premium of ₹ 2. Find the number of shares she purchased. complete the given activity to get the answer.

Solution : FV = ₹ 10, Premium = ₹ 2,.

∴ MV = FV + ☐ = ☐ + ☐ = ☐

∴ Number of shares = $\dfrac{\text{Total investment}}{\text{MV}}$ = $\dfrac{12000}{\boxed{}}$ = ☐ shares

Ans : Smita has purchased ☐ shares.

Ex. (6) If 50 shares of FV ₹10 were purchased for MV of ₹ 25. Company declared 30% dividend on the shares then find (1) Sum investment (2) Dividend received (3) Rate of return.

Solution : FV = ₹10, MV = ₹25, Number of shares = 50.

(1) ∴ Sum investment = 25 × 50 = ₹1250.

(2) Dividend per share = $10 \times \dfrac{30}{100}$ = ₹3

∴ Total dividend received = 50 × 3 = ₹ 150.

(3) Rate of return = $\dfrac{\text{Dividend income}}{\text{Sum invested}} \times 100$

$$= \frac{150}{1250} \times 100 = 12\%$$

Ans : (1) Sum invested is ₹ 1250 (2) Dividend received is ₹ 150

(3) Rate of return is 12%.

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Practice Set 4.3

</div>

(1) Complete the following table by writting suitable numbers and words.

Sr.No	FV	Share is at	MV
(1)	₹ 100	par	. . .
(2)	. . .	premium ₹ 500	₹ 575
(3)	₹ 10	. . .	₹ 5

(2) Mr.Amol purchased 50 shares of Face Value ₹ 100 when the Market value of the share was ₹ 80. Company had given 20% dividend. Find the rate of return on investment.

(3) Joseph purchased following shares, Find his total investment.

Company A : 200 shares,   FV = ₹ 2   Premium = ₹ 18.

Company B : 45 shares,   MV = ₹ 500

Company C : 1 share,   MV = ₹ 10,540.

(4) Smt. Deshpande purchased shares of FV ₹ 5 at a premium of ₹ 20. How many shares will she get for ₹ 20,000 ?

(5) Shri Shantilal has purchased 150 shares of FV ₹ 100, for MV of ₹ 120. Company has paid dividend at 7%. Find the rate of return on his investment.

(6) If the face value of both the shares is same, then which investment out of the following is more profitable ?

Company A : dividend 16%, MV = ₹ 80, Company B : dividend 20%, MV = ₹ 120.
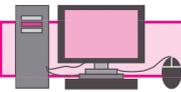
ICT Tools or Links.

Select any five shares of your choice, find their Face Values and Market Values using internet or TV or news papers. Draw the joint bar diagram and compare the difference in FV and MV of each share. Take both the types ▲, ▼ of shares.

Let's learn.

Brokerage and taxes on share trading

Brokerage : We directly can't go to the stock market and buy or sell shares, only the registered members or organization (agency) of the stock market can buy or sell on our behalf. These members are called **'Share Brokers'**. For catering the service of buying and selling of shares they charge some amount which is called **'Brokerage'** Brokerage is paid on the Market value of the share.

Ex. (1) Suppose if the face value of the share is ₹ 100 and market value is ₹ 150. Let the rate of brokerage be 0.5%. What amount should one pay for purchasing 100 such shares ? What amount should one receive after selling 100 such shares ?

Situation (I) At the time of buying shares:
Buying price of 1 share = MV + Brokerage

$$= 150 + 0.5\% \text{ of } 150$$
$$= 150 + 0.75$$
$$= ₹ 150.75$$

If someone purchases 100 such shares the total cost is $100 \times 150.75 = ₹ 15075$. Here ₹ 15000 is the share price and ₹ 75 is the brokerage paid.

Situation (II) At the time of selling shares.
Selling price per share = MV - Brokerage

$$= 150 - 5\% \text{ of } 150 = 150 - 0.75$$
$$= ₹ 149.25.$$

If someone sells 100 such shares, he will get,

100 × 150 - ₹ 75 = ₹ 14925 after selling 100 such shares.

Project I : Visit the office of a share broker or agency in your area. Collect the information of brokerage charges, other charges and facilities given to the investors and compare.

Project II : Obtain a statement of 'Demat Account' and 'Trading Account'. Consult a share broker or elders in the house or use internet. Try to learn all the terms in the statement. Discuss with your friends in the class.

For more information :

Every broker is registered and governed by SEBI (Securities and Exchange Board of India) Act 1992.

For keeping records of shares, bonds, mutual funds one must have Demat account (Dematerialised Account). For sale and purchase of shares, a trading account is a must. These accounts can be opened with banks or share brokers. They are known as DP-Depository Participants. These DPs are under the control of two main depositories namely NSDL and CDSL. Demat account is same as bank saving account where shares bought are credited and shares sold are debited just like bank pass book. The statement of holding is given to the account holder with nominal charges when requested. The shares held in Demat A/c are in electronic form. Saving account is to be linked with these two accounts so that the money can be transfered as and when required. In the same way money gets credited when shares are sold. For opening these accounts share broker or bank representative gives guidance.

Let's learn.

GST on Brokerage Services

Share brokers provide services for purchase and sale of shares for their clients. These services are charged under GST. Rate of GST is 18% on brokerage. You can find the SAC for brokerage services.

Note :- For the safety of the investors, there are other nominal charges besides GST on brokerage. These are Security Transaction Tax (STT), SEBI charge, stamp duty etc. Here we will only consider GST on brokerage.

Ex. (2) As per Ex. (1) suppose a person has paid ₹ 15,075 for buying 100 shares. In that ₹ 75 is the brokerage. So the buyer has to pay 18% GST on ₹ 75. Let us find the amount of GST he paid to the broker and prepare the contract note.

Solution : GST = 18% of 75 = $\dfrac{18}{100} \times 75$ = ₹ 13.50.

For the above share trading the contract note is as follows. (B means Buy)

No. of shares	MV	Total price	brokerage 0.5%	CGST 9% on brokerage	SGST 9% on brokerage	Total value.
100 (B)	150	₹ 15000	₹ 75	₹ 6.75	₹ 6.75	₹ 15088.50

Ex. (3) Bashirkhan purchased 100 shares of MV ₹ 40. Brokerage paid at the rate of 0.5% and rate of GST on brokerage is 18%. Find the total amount he paid for the share purchase.

Solution : Value of 100 shares = 40 × 100 = ₹ 4000.

$\qquad$ Brokerage per share = $\dfrac{0.5}{100} \times 40$ = ₹ 0.20.

$\therefore$ Cost of one share = MV + Brokerage

$$= 40 + 0.20 = ₹ \ 40.20.$$

$\therefore$ Cost of 100 shares = 40.20 × 100 = ₹ 4020

$\therefore$ Brokerage on 100 share = 0.20 × 100 = ₹ 20

$\therefore$ GST = $\dfrac{18}{100} \times 20$

$\qquad$ = ₹ 3.60.

Ans. : Bashirkhan paid ₹ 4020 + ₹ 3.60 = ₹ 4023.60 for 100 shares.

Ex. (4) Pankajrao invested ₹ 1,25,295 in shares of FV ₹ 10 when MV is ₹ 125. Rate of brokerage is 0.2% and GST is 18%. Then find (1) How many shares were purchased. (2) the amount of brokerage paid and (3) GST paid for the trading.

Solution : Sum invested = ₹ 1,25,250, brokerage = 0.2%, GST rate = 18%

$\qquad \therefore$ Brokerage per share = 125 × $\dfrac{0.2}{100}$ = ₹ 0.25.

$\qquad$ GST per share on brokerage = 18% of 0.25 = ₹ 0.045

$\qquad \therefore$ Cost of 1 share = MV + Brokerage + GST

$$= 125 + 0.25 + 0.045 = ₹ \ 125.295.$$

$$\therefore \text{ No. of shares } = \frac{125250}{125.25} = 1000$$

Total brokerage = brokerage per share × No. of shares

∴ Total brokerage = 0.25 × 1000 = ₹ 250.

Total GST =1000 × 0.045 = ₹ 45.

Ans. (1) 1000 shares were purchased.

(2) Brokerage paid was ₹ 250.

(3) GST paid was ₹ 45.

Ex. (5) Nalinitai invested ₹ 6024 in the shares of FV ₹ 10 when the Market Value was ₹ 60. She sold all the shares at MV of ₹ 50 after taking 60% dividend. She paid 0.4% brokerage at each stage of transactions. What was the total gain or loss in this transaction ?

Solution : Rate of GST is not given in the example, so it is not considered.

Shares Purchased : FV = ₹ 10, MV = ₹ 60

Brokerage per share $= \frac{0.4}{100} × 60 = ₹ \boxed{}$

∴ Cost of one share $= 60 + 0.24 = ₹ \boxed{}$

∴ Number of shares $= \frac{6024}{60.24} = 100$

Shares sold : FV ₹ 10, MV = ₹ 50

∴ Brokerage per share $= \frac{0.4}{100} × 50 = ₹ \ 0.20$

∴ Selling price per share $= 50 - 0.20 = ₹ \boxed{}$

∴ Selling price of 100 shares $= 100 × 49.80 = ₹ \boxed{}$

Dividend received 60%

∴ Dividend per share $= \frac{60}{100} × 10 = ₹ \ 6$

∴ Dividend on 100shares $= 6 × 100 = ₹ \boxed{}$

∴ Nalinitai's income $= \boxed{} + \boxed{} = ₹ \ 5580$

Sum invested = ₹ 6024

∴ Loss $= \boxed{} - \boxed{} = ₹ \boxed{}$

Ans. Nalinitai's loss is ₹ 444

Activity : In example (5) if GST was paid at 18% on brokerage, then the loss is ₹ 451.92. Verify whether you get the same answer.

Let's learn.

Mutual Fund - MF

We have learnt that a group of persons come together to form a company. They raise capital from the society by issuing shares. If company performs well, then the investors of the company get benefits in terms of dividend, bonus shares and increase in the market value gives more profit on investments. Company's market capitalization rises. All this totality helps for the progress of the country. In short, principle of sociology 'together we can progress' works here. But every coin has two sides. sometimes it might happen that instead of profit an investor may incur a loss. Can we reduce this loss ? Is there a way to reduce the risk in investments ? Yes, to overcome this more people invest in Mutual Funds.

In Mutual Fund, many investors with common objectives give their money to the professional experts. They not only invest in one type of shares but also invest in various other schemes. As a result, investment is diversified which reduces risk factor and total dividend or profit is divided equally among the investors. How to invest in Mutual Fund ? What is the rate of return ? What is the locking period ? What are the different types of investment schemes ? All these questions could be answered by a Financial advisor or financial planner.

> You may have heard or read this sentence that, 'Investments in Mutual Funds are subject to Market risks. Read all scheme related documents carefully before invesing.' Interpret the meaning correctly. Sometimes instead of profit, investment in Mutual Fund might give loss which investors have to bear.

Mutual Fund is a professionally managed investment scheme, usually run by an AMC i.e. Asset Management Company. They invest the money given by the investors in different schemes e.g. equity fund (in shares), debt fund (in debentures, bonds etc.) or balanced funds as per the investor's choice.

As we get 'shares' for the investment in sharemarket, we get '**units**' when we invest in mutual fund.

The market value of 'a unit' is called '**NAV**' (Net Asset Value)

NAV of one unit × Number of units = Total fund value.

Note : As the market value of share changes frequently NAV of a unit also changes. One can redeem the units when needed.

Investments in FDs of nationalised bank or Indian Postal services are more secured and safe, in comparison with other investments, but the rate of return is low. It hardly helps to overcome the rate of inflation. One must remember always that if the money is invested wisely it generates more money. For this the knowledge of financial planning is of great help.

Investments in shares and mutual funds should be made carefully because risk and returns always go hand in hand. So the habit of regular and deep study is the only key.

Systematic Investment Plan

Suppose, one does not want to invest a big amount at once, then one could invest small amounts at regular time intervals e.g. ₹ 500 per month could be invested in mutual fund. Investment could be done monthly or quarterly. This way of investment is called SIP. SIP develops discipline of savings. SIP is a good option which in long term can achieve one's financial goals. Investment in mutual funds through SIP for a long term is beneficial. It protects investor from market fluctuations. One should invest in mutual fund for minimum of 3 to 5 years to get better returns and it is best if investment is for 10 to 15 years.

Benefits of Mutual Funds

- Professional fund managers.
- Diversifications of funds.
- Transparency and sufficiently safe investment.
- Liquidity - redemption of units can be done.
- Limited risks.
- Advantage of long term and short term gain.
- Investments in funds like ELSS are admissible for deduction under section 80C of income tax.

✍✍✍ Solved Examples ✍✍✍

Ex. (1) If the total value of the mutual fund scheme is ₹ 200 crores and 8 crore units are issued then find the NAV of one unit.

Solution : NAV = ₹ 200 crore / 8 crore units = ₹ 25 per unit.

Ex. (2) If NAV of one unit is ₹ 25, then how many units will be alloted for the investment of ₹ 10,000 ?

Solution : Number of units = sum invested / NAV = 10,000/25 = 400 units.

Practice Set 4.4

1. Market value of a share is ₹ 200. If the brokerage rate is 0.3% then find the purchase value of the share.

2. A share is sold for the market value of ₹ 1000. Brokerage is paid at the rate of 0.1%. What is the amount received after the sale ?

3. Fill in the blanks given in the contract note of sale-purchase of shares. (B - buy S - sell)

No. of shares	MV of shares	Total value	Brokerage 0.2%	9% CGST on brokerage	9% SGST on brokerage	Total value of shares
100 B	₹ 45					
75 S	₹ 200					

4. Smt. Desai sold shares of face value ₹ 100 when the market value was ₹ 50 and received ₹ 4988.20. She paid brokerage 0.2% and GST on brokerage 18%, then how many shares did she sell ?

5. Mr. D'souza purchased 200 shares of FV ₹ 50 at a premium of ₹ 100. He received 50% dividend on the shares. After receiving the dividend he sold 100 shares at a discount of ₹ 10 and remaining shares were sold at a premium of ₹ 75. For each trade he paid the brokerage of ₹ 20. Find whether Mr. D'souza gained or incurred a loss ? by how much ?
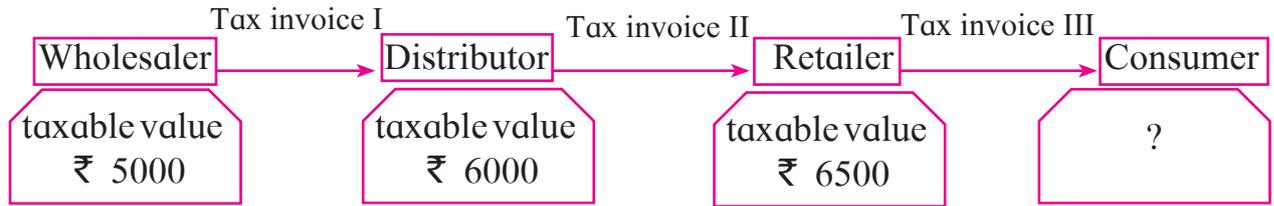
Problem Set 4A

1. Write the correct alternative for each of the following.

(1) Rate of GST on essential commodities is …

(A) 5%   (B) 12%   (C) 0%   (D) 18%

(2) The tax levied by the central government for trading within state is…

(A) IGST   (B) CGST   (C) SGST   (D) UTGST

(3) GST system was introduced in our country from …

(A) 31st March 2017   (B) 1st April 2017

(C) 1st January 2017   (D) 1st July 2017

(4) The rate of GST on stainless steel utensils is 18%, then the rate of State GST is . . .

(A) 18%   (B) 9%   (C) 36%   (D) 0.9%

(5) In the format of GSTIN there are … alpha-numerals.

(A) 15   (B) 10   (C) 16   (D) 9

(6) When a registered dealer sells goods to another registered dealer under GST, then this trading is termed as . . .

   (A) BB   (B) B2B   (C) BC   (D) B2C

2. A dealer has given 10% discount on a showpiece of ₹ 25,000. GST of 28% was charged on the discounted price. Find the total amount shown in the tax invoice. What is the amount of CGST and SGST ?

3. A ready-made garment shopkeeper gives 5% discount on the dress of ₹ 1000 and charges 5% GST on the remaining amount, then what is the purchase price of the dress for the customer ?

4. A trader from Surat, Gujarat sold cotton clothes to a trader in Rajkot, Gujarat. The taxable value of cotton clothes is ₹ 2.5 lacs. What is the amount of GST at 5% paid by the trader in Rajkot ?

5. Smt. Malhotra purchased solar panels for the taxable value of ₹ 85,000. She sold them for ₹ 90,000. The rate of GST is 5%. Find the ITC of Smt. Malhotra. What is the amount of GST payable by her ?

6. A company provided Z-security services for the taxable value of ₹ 64,500. Rate of GST is 18%. Company had paid GST of ₹ 1550 for laundry services and uniforms etc. What is the amount of ITC (input Tax Credit) ? Find the amount of CGST and SGST payable by the company.

7. A dealer supplied Walky-Talky set of ₹ 84,000 (with GST) to police control room. Rate of GST is 12%. Find the amount of state and central GST charged by the dealer. Also find the taxable value of the set.

8.★ A wholesaler purchased electric goods for the taxable amount of ₹ 1,50,000. He sold it to the retailer for the taxable amount of ₹ 1,80,000. Retailer sold it to the customer for the taxable amount of ₹ 2,20,000. Rate of GST is 18%. Show the computation of GST in tax invoices of sales. Also find the payable CGST and payable SGST for wholesaler and retailer.

9.★ Anna Patil (Thane, Maharashtra) supplied vacuum cleaner to a shopkeeper in Vasai (Mumbai) for the taxable value of ₹ 14,000, and GST rate of 28%. Shopkeeper sold it to the customer at the same GST rate for ₹ 16,800 (taxable value) Find the following -

   (1) Amount of CGST and SGST shown in the tax invoice issued by Anna Patil.
   (2) Amount of CGST and SGST charged by the shopkeeper in Vasai.
   (3) What is the CGST and SGST payable by shopkeeper in Vasai at the time of filing the return.

10.★ For the given trading chain prepare the tax invoice I, II, III. GST at the rate of 12% was charged for the article supplied.

Tax invoice I   Tax invoice II   Tax invoice III

| Wholesaler | → | Distributor | → | Retailer | → | Consumer |

taxable value ₹ 5000   taxable value ₹ 6000   taxable value ₹ 6500   ?

(1) Prepare the statement of GST payable under each head by the wholesaler, distributor and retailer at the time of filing the return to the government.

(2) At the end what amount is paid by the consumer ?

(3) Write which of the invoices issued are B2B and B2C ?

Problem Set 4B

1. Write the correct alternative for the following questions.

   (1) If the Face Value of a share is ₹ 100 and Market value is ₹ 75, then which of the following statements is correct ?

   (A) The share is at premium of ₹ 175 (B) The share is at discount of ₹ 25

   (C) The share is at premium of ₹ 25 (D) The share is at discount of ₹ 75

   (2) What is the amount of dividend received per share of face value ₹ 10 and dividend declared is 50%.

   (A) ₹ 50 (B) ₹ 5 (C) ₹ 500 (D) ₹ 100

   (3) The NAV of a unit in mutual fund scheme is ₹ 10.65 then find the amount required to buy 500 such units.

   (A) 5325 (B) 5235 (C) 532500 (D) 53250

   (4) Rate of GST on brokerage is . . .

   (A) 5% (B) 12% (C) 18% (D) 28%

   (5) To find the cost of one share at the time of buying the amount of Brokerage and GST is to be . . . the MV of share .

   (A) added to (B) substracted from

   (C) Multiplied with (D) divided by

2. Find the purchase price of a share of FV ₹ 100 if it is at premium of ₹ 30. The brokerage rate is 0.3%.

3. Prashant bought 50 shares of FV ₹ 100, having MV ₹ 180. Company gave 40% dividend on the shares. Find the rate of return on investment.

4. Find the amount received when 300 shares of FV ₹ 100, were sold at a discount of ₹ 30.

5. Find the number of shares received when ₹ 60,000 was invested in the shares of FV ₹ 100 and MV ₹ 120.

6. Smt. Mita Agrawal invested ₹ 10,200 when MV of the share is ₹ 100. She sold 60 shares when the MV was ₹ 125 and sold remaining shares when the MV was ₹ 90. She paid 0.1% brokerage for each trading. Find whether she made profit or loss ? and how much ?

7. Market value of shares and dividend declared by the two companies is given below. Face Value is same and it is ₹ 100 for both the shares. Investment in which company is more profitable ?
   (1) Company A - ₹ 132 , 12%   (2) Company B - ₹ 144, 16%

8.★ Shri. Aditya Sanghavi invested ₹ 50,118 in shares of FV ₹ 100, when the market value is ₹ 50. Rate of brokerage is 0.2% and Rate of GST on brokerage is 18%, then How many shares were purchased for ₹ 50,118 ?

9.★ Shri. Batliwala sold shares of ₹ 30,350 and purchased shares of ₹ 69,650 in a day. He paid brokerage at the rate of 0.1% on sale and purchase. 18% GST was charged on brokerage. Find his total expenditure on brokerage and tax.

10.★ Smt. Aruna Thakkar purchased 100 shares of FV 100 when the MV is ₹ 1200. She paid brokerage at the rate of 0.3% and 18% GST on brokerage. Find the following -

   (1) Net amount paid for 100 shares.

   (2) Brokerage paid on sum invested.

   (3) GST paid on brokerage.

   (4) Total amount paid for 100 shares.

11.★ Smt. Anagha Doshi purchased 22 shares of FV ₹ 100 for Market Value of ₹ 660. Find the sum invested. After taking 20% dividend, she sold all the shares when market value was ₹ 650. She paid 0.1% brokerage for each trading done. Find the percent of profit or loss in the share trading.

   (Write your answer to the nearest integer.)

5 ▶ Probability

- Probability : Introduction
- Sample space and event
- Random experiment and its outcome
- Probability of an event

Let's discuss.

Teacher : Friends, this box contains folded chits. The number of chits is exactly the same as the number of students in our class. Each student should pick one chit. Names of different plants are written on the chits. No two chits bear the same name of the plant. Let us see who gets the chit having the name 'Basil'. Make a line in the order of your roll numbers. No one will unfold the chit until the last student takes his chit.

Aruna : Sir, I am the first one in a line, but I do not want to pick a chit first, as the possibility of getting 'basil' chit from all the chits is very low.

Zarina : Sir, I am the last student in the row, I do not want to pick the chit at last as the chit containing the name 'basil' will most likely be picked up by some one else before my turn.

The first and the last student feel that for each of them, the possibility of getting the chit having the name 'basil' is very low. The above conversation indicates the thinking of less or more possibility.

We use the following words to express the possibility in our daily conversation.

- Probable
- sure
- may be
- nearly
- impossible
- 50 – 50

Read the following statements regarding predictions (possibilities for the future).
- Most probably the rain will start from today.
- The inflation is likely to rise.
- It is impossible to defeat Indian cricket team in the next match.
- I will surely get first class.
- There is no possibility of Polio infection if a child is given the polio vaccine in time.

The adjoining picture shows a 'toss' before a cricket match.

What are the possibilities ?

☐ or ☐

So here there are ☐ possibilities.

Activity 1 : Let each student in the class toss a coin once. What will you get?
(Teacher writes the observations on the board in a table.)

Possibilities	(H)	(T)
Number of students	. . .	. . .

Activity 2 : Ask each student to toss the same coin twice. What are the possibilities?

Possibilities	H H	HT	TH	TT
Number of students				

Activity 3 : Now throw a die, once. What are the different possibilities of getting dots on the upper face ?

Each of these is a possible result of throwing a die.

Let's learn.

Random Experiment

The experiment in which all possible results are known in advance but none of them can be predicted with certainty and there is equal possibility for each result is known as a 'Random experiment'.

For example, Tossing a coin, throwing a die, picking a card from a set of cards bearing numbers from 1 to 50, picking a card from a pack of well shuffled playing cards, etc.

Result of a random experiment is known as an 'Outcome'.

Ex. (1) In a random experiment of tossing a coin - there are only two outcomes.
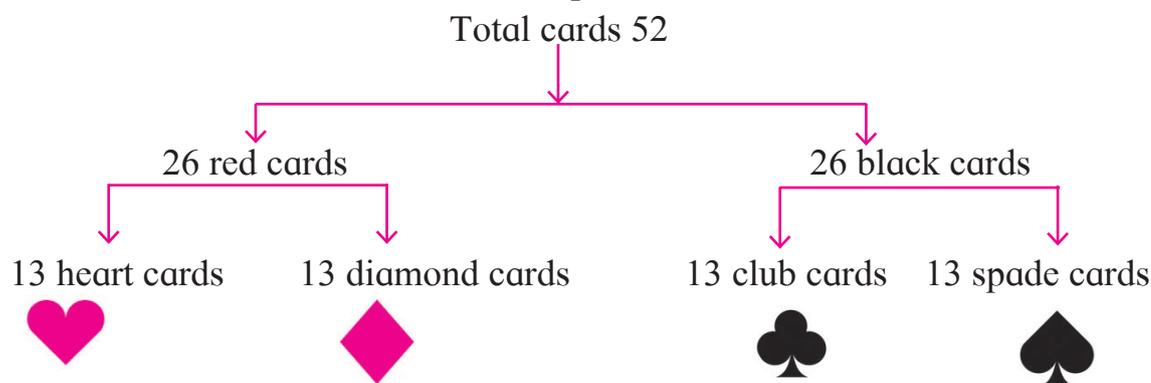
Head (H) or Tail (T)

(2) In a random experiment of throwing a die, there are 6 outcomes, according to the number of dots on the six faces of the die.

1 or 2 or 3 or 4 or 5 or 6.

(3) In a random experiment of picking a card bearing numbers from 1 to 50, there are 50 outcomes.

(4) A card is drawn randomly from a pack of well shuffled playing cards.

There are 52 cards in a pack as shown below.

Total cards 52

26 red cards | 26 black cards

13 heart cards   13 diamond cards   13 club cards   13 spade cards

In a pack of playing cards there are 4 sets, namely heart, diamond, club and spade. In each set there are 13 cards as King, Queen, Jack, 10, 9, 8, 7, 6, 5, 4, 3, 2 and Ace.

King, Queen and Jack are known as face cards. In each pack of cards there are 4 cards of king, 4 cards of Queen and 4 cards of Jack. Thus total face cards are 12.

Ace of spade   Ace of heart   Ace of club   Ace of diamond
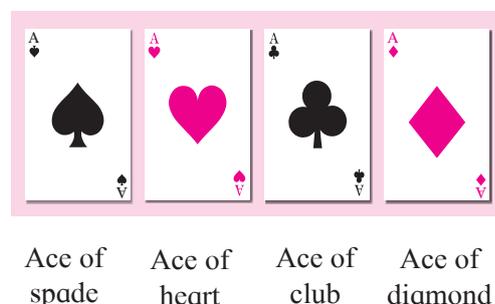
Equally Likely Outcomes

If a die is thrown, any of the numbers from 1, 2, 3, 4, 5, 6 may appear on the upper face. It means that each number is equally likely to occur. However, if a die is so formed that a particular face come up most often, then that die is biased. In this case the outcomes are not likely to occur equally.

Here, we assume that objects used for random experiments are fair or unbiased.

A given number of outcomes are said to be equally likely if none of them occurs

in preference to others. For example if a coin is tossed, possibilities of getting head or tail are equal. A die, having numbers from 1 to 6 on its different faces, is thrown. Check the possibility of getting one of the numbers. Here all the outcomes are eqully likely.

Practice Set 5.1

1. How many possibilities are there in each of the following?

   (1) Vanita knows the following sites in Maharashtra. She is planning to visit one of them in her summer vacation.

   Ajintha, Mahabaleshwar, Lonar Sarovar, Tadoba wild life sanctuary, Amboli, Raigad, Matheran, Anandavan.

   (2) Any day of a week is to be selected randomly.

   (3) Select one card from the pack of 52 cards.

   (4) One number from 10 to 20 is written on each card. Select one card randomly.

Let's think.

In which of the following experiments possibility of expected outcome is more?

   (1) Getting 1 on the upper face when a die is thrown.

   (2) Getting head by tossing a coin.

Let's learn.

Sample Space

The set of all possible outcomes of a random experiment is called the sample space. It is denoted by 'S' or 'Ω' (A greek letter 'Omega'). Each element of sample space is called a 'sample point'. The number of elements in the set 'S' is denoted by n(S). If n(S) is finite, then the sample space is said to be a finite sample space.

Following are some examples of finite sample spaces.

S. No.	Random experiment	Sample space	Number of sample points in S
1	One coin is tossed	S = {H, T)	n(S) = 2
2	Two coins are tossed	S = { HH, HT, TH, TT}	n(S) = ☐
3	Three coins are tossed	S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}	n(S) = 8
4	A die is thrown	S = {1, 2, 3, 4, 5, 6}	n(S) = ☐
5	Two dice are thrown	S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}	n(S) = 36
6	A card is drawn from a pack bearing numbers from 1 to 25	S = {1, 2, 3, 4, ..., 25}	n(S) = ☐
7	A card is drawn from a well shuffled pack of 52 playing cards.	Diamond : Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King Spade : Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King Heart : Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King Club : Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King	n(S) = 52

Let's remember!

(i) The sample space for a coin tossed twice is the same as that of two coins tossed simultaneously. The same is true for three coins.

(ii) The sample space for a die rolled twice is the same as two dice rolled simultaneously.
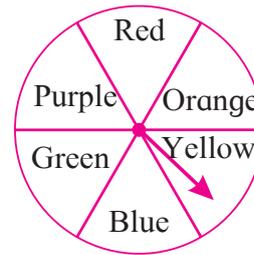
Practice Set 5.2

(1) For each of the following experiments write sample space 'S' and number of sample points n(S).

(1) One coin and one die are thrown simultaneously.

(2) Two digit numbers are formed using digits 2, 3 and 5 without repeating a

digits.

2. The arrow is rotated and it stops randomly on the disc. Find out on which colour it may stop.

MARCH - 2019						
M	T	W	T	F	S	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

3. In the month of March 2019, find the days on which the date is a multiple of 5. (see the given page of the calender)

4. Form a 'Road safety commitee' of two, from 2 boys (B_1, B_2) and 2 girls (G_1, G_2). Complete the following activity to write the sample space.

(a) Committee of 2 boys = [　　] (b) Committee of 2 girls = [　　]

(c) Committee of one boy and one girl = $\boxed{B_1\ G_1}$ [　] [　] [　]

∴ Sample space = {..., ..., ..., ..., ..., ...}

Let's learn.

Event

The outcomes satisfying particular condition are called favourable outcomes.

A set of favourable outcomes of a given sample space is an '**event**'. Event is a subset of the sample space.

Events are generally denoted by capital letteres A, B, C, D etc. For example, if two coins are tossed and A is the event of getting at least one tail, then the favourable outcomes are as follows.

$$A = \{TT, TH, HT\}$$

The number of elements in the event A is denoted by $n(A)$. Here $n(A) = 3$.

For more information

Types of event.
(i) Certain event/Sure event
(ii) Impossible event
(iii) Simple/Elementary event
(iv) Complement of an event
(v) Mutually exclusive events
(vi) Exhaustive event

Ex. (1) Two coins are tossed simultaneously. Write the sample space (S) and number of sample points n(S). Also write the following events in the set form and write the number of sample points in each event.

(i) Condition for event A : to get at least one tail.

(ii) Condition for event B : to get only one head.

(iii) Condition for event C : to get at most one tail.

(iv) Condition for event D : to get no head.

Solution : If two coins are tossed simultaneously,

$\qquad$ S = {HH, HT, TH, TT} $\qquad$ n(S) = 4

(i) Condition for event A : at least one head.

$\qquad$ A = {HH, HT, TH} $\qquad$ n(A) = 3

(ii) Condition for event B : only one head.

$\qquad$ B = { HT, TH} $\qquad$ n(B) = 2

(iii) Condition for event C : at most one tail.

$\qquad$ C = {HH, HT, TH} $\qquad$ n(C) = 3

(iv) Condition for event D : No head.

$\qquad$ D = {TT} $\qquad$ n(D) = 1

Ex. (2) A bag contains 50 cards. Each card bears only one number from 1 to 50. One card is drawn at randomfrom the bag. Write the sample space. Also write the events A, B and find the number of sample points in them.

(i) Condition for event A : the number on the card is divisible by 6.

(ii) Condition for event B : the number on the card is a complete square.

Solution : S = {1, 2, 3, . . . 49, 50}, n(S) = 50

(i) Condition for event A : number is divisible by 6.

$\qquad$ A = {6, 12, 18, 24, 30, 36, 42, 48}   n(A) = 8

(ii) Condition for event B : the number on the card is a complete square.

$\qquad$ B = {1, 4, 9, 16, 25, 36, 49}   n(B) = 7

Ex. (3) A sanitation committee of 2 members is to be formed from 3 boys and 2 girls. Write sample space 'S' and number of sample points n(S). Also write the following events in set form and number of sample points in the event.

(i) Condition for event A : at least one girl must be a member of the committee.

(ii) Condition for event B : Committee must be of one boy and one girl.

(iii) Condition for event C : Committee must be of boys only.

(iv) Condition for event D : At the most one girl should be a member of the committee.

Solution : Suppose B_1, B_2, B_3 are three boys and G_1, G_2 are two girls

Out of these boys and girls, a sanitation committee of two members is to be formed.

$\therefore$ S = {$B_1B_2, B_1B_3, B_2B_3, B_1G_1, B_1G_2, B_2G_1, B_2G_2, B_3G_1, B_3G_2, G_1G_2$} $\therefore$ n(S) = 10

(i) Condition for event A is that at least one girl should be in the committee.

A = {$B_1G_1, B_1G_2, B_2G_1, B_2G_2, B_3G_1, B_3G_2, G_1G_2$} $\therefore$ n(A) = 7

(ii) Condition for event B is that one boy and one girl should be there in the committee.

B = {$B_1G_1, B_1G_2, B_2G_1, B_2G_2, B_3G_1, B_3G_2$} $\therefore$ n(B) = 6

(iii) Condition for event C is that there should be only boys in the committee.

C = {B_1B_2, B_1B_3, B_2B_3}   n(C) = 3

(iv) Condition for event D is that there can be at most one girl in the committee.

D = {$B_1B_2, B_1B_3, B_2B_3, B_1G_1, B_1G_2, B_2G_1, B_2G_2, B_3G_1, B_3G_2$} $\therefore$ n(D) = 9

Ex. (4) Two dice are rolled, write the sample space 'S' and number of sample points n(S). Also write events and number of sample points in the event according to the given condition.

(i) Sum of the digits on upper face is a prime number.

(ii) Sum of the digits on the upper face is multiple of 5.

(iii) Sum of the digits on the upper face is 25.

(iv) Digit on the upper face of the first die is less than the digit on the second die.

Solution : Sample space,

S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),

(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),

(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),

(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),

(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}   n(S) = 36

(i) Event E : The sum of the digits on the upper face is a prime number.

E = {(1, 1), (1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4),

(4, 1), (4, 3), (5, 2), (5, 6), (6, 1), (6, 5)}   ∴ n(E) = 15

(ii) Event F : The sum of the digits on the upper face is a multiple of 5.

F = { (1, 4), (2, 3), (3, 2), (4, 1), (4, 6), (5, 5), (6, 4) }   ∴ n(F) = 7

(iii) Event G : The sum of the digits on the upper face is 25.

G = { } = φ   ∴ n (G) = 0

(iv) Event H : The number on upper face of first die is less than the digit on second die.

H = {(1, 2) (1, 3) (1, 4) (1, 5) (1, 6) (2, 3) (2, 4) (2, 5) (2, 6)

(3, 4) (3, 5) (3, 6) (4, 5) (4, 6) (5, 6)}   ∴ n(H) = 15

Practice Set 5.3

1. Write sample space 'S' and number of sample point n(S) for each of the following experiments. Also write events A, B, C in the set form and write n(A), n(B), n(C).

(1) One die is rolled,

Event A : Even number on the upper face.

Event B : Odd number on the upper face.

Event C : Prime number on the upper face.

(2) Two dice are rolled simultaneously,

Event A : The sum of the digits on upper faces is a multiple of 6.

Event B : The sum of the digits on the upper faces is minimum 10.

Event C : The same digit on both the upper faces.

(3) Three coins are tossed simultaneously.

Condition for event A : To get at least two heads.

Condition for event B : To get no head.

Condition for event C : To get head on the second coin.

(4) Two digit numbers are formed using digits 0, 1, 2, 3, 4, 5 without repetition of the digits.

Condition for event A : The number formed is even

Condition for event B : The number formed is divisible by 3.

Condition for event C : The number formed is greater than 50.

(5) From three men and two women, environment committee of two persons is to be formed.

Condition for event A : There must be at least one woman member.

Condition for event B : One man, one woman committee to be formed.

Condition for event C : There should not be a woman member.

(6) One coin and one die are thrown simultaneously.

Condition for event A : To get head and an odd number.

Condition for event B : To get a head or tail and an even number.

Condition for event C : Number on the upper face is greater than 7 and tail on the coin.

Let's learn.

Probability of an event

Let us think of a simple experiment. A bag contains 4 balls of the same size. Three of them are white and the fourth is black. You are supposed to pick one ball at random without seeing it. Then obviously, possibility of getting a white ball is more.

In Mathematical language, when possibility of an expected event is expressed in number, it is called 'Probability' . It is expressed as a fraction or percentage using the following formula.

For a random experiment, if sample space is 'S' and 'A' is an expected event then probability of 'A' is P(A). It is given by following formula.

$$P(A) = \frac{\text{Number of sample points in event A}}{\text{Number of sample points in sample spaces}} = \frac{n(A)}{n(S)}$$

In the above experiment, getting a white ball is event A. As there are three white balls n(A) = 3, As the number of balls is 4, n(S) = 4

∴ probability of getting a white ball is, $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{3}{4}$.

Similarly, if getting black ball is event B, then n(B) = 1 ∴ $P(B) = \dfrac{n(B)}{n(S)} = \dfrac{1}{4}$.

ℬℬℬ Solved Examples ℬℬℬ

Ex. (1) Find the probability of the following, when one coin is tossed.

      (i) getting head   (ii) getting tail

Solution : Let 'S' be the sample space.

      S = {H, T}   n(S) = 2

(i) Let event A be getting head

      A = {H}   ∴ n(A) = 1

      $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{1}{2}$

(ii) Let event B be getting tail

      B = {T}   ∴ n(B) = 1

      $P(B) = \dfrac{n(B)}{n(S)} = \dfrac{1}{2}$

Ex. (2) If one die is rolled then find the probability of each of the following events.

      (i) Number on the upper face is prime

      (ii) Number on the upper face is even.

Solution : 'S' is the sample space.

      S = {1, 2, 3, 4, 5, 6}   ∴ n(S) = 6

      (i) Event A : Prime number on the upper face.

      A = {2, 3, 5}   ∴ n(A) = 3

        $P(A) = \dfrac{n(A)}{n(S)}$

      ∴ $P(A) = \dfrac{3}{6} = \dfrac{1}{2}$

(ii) Event B : Even number on the upper face.

$$B = \{2, 4, 6\} \qquad \therefore n(B) = 3$$

$$P(B) = \frac{n(B)}{n(S)}$$

$$\therefore P(B) = \frac{3}{6} = \frac{1}{2}$$

Ex. (3) A card is drawn from a well shuffled pack of 52 playing cards. Find the probability of each event. The card drawn is (i) a red card   (ii) a face card

Solution : 'S' is the sample space. $\therefore n(S) = 52$

Event A : Card drawn is a red card.

Total red cards = 13 hearts + 13 diamonds = 26

$$\therefore n(A) = 26$$

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{26}{52} = \frac{1}{2}$$

Event B : Card drawn is a face card.

Total face cards = 12 $\therefore n(B) = 12$

$$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{12}{52} = \frac{3}{13}$$

Ex. (4) A box contains 5 strawberry chocolates, 6 coffee chocolates and 2 peppermint chocolates. Find the probability of each of the following events, if one of the chocolates is picked from the box at random. (i) it is a coffee chocolate.

(ii) it is a peppermint chocolate.

Solution : Sample space is 'S' and n(S) = 5 + 6 + 2 = 13

Event A : it is a coffee chocolate

$$\therefore \quad n(A) = 6$$

$$\therefore \quad P(A) = \frac{n(A)}{n(S)} = \frac{6}{13}$$

Event B : it is a peppermint chocolate

$$\therefore \quad n(B) = 2$$

$$\therefore \quad P(B) = \frac{n(B)}{n(S)} = \frac{2}{13}$$

Practice Set 5.4

1. If two coins are tossed, find the probability of the following events.

   (1) Getting at least one head.   (2) Getting no head.

2. If two dice are rolled simultaneously, find the probability of the following events.

   (1) The sum of the digits on the upper faces is at least 10.

   (2) The sum of the digits on the upper faces is 33.

   (3) The digit on the first die is greater than the digit on second die.

3. There are 15 tickets in a box, each bearing one of the numbers from 1 to 15. One ticket is drawn at random from the box. Find the probability of event that the ticket drawn -

   (1) shows an even number.   (2) shows a number which is a multiple of 5.

4. A two digit number is formed with digits 2, 3, 5, 7, 9 without repetition. What is the probability that the number formed is

   (1) an odd number ?   (2) a multiple of 5 ?

5. A card is drawn at random from a pack of well shuffled 52 playing cards. Find the probability that the card drawn is -

   (1) an ace.   (2) a spade.

Problem Set 5

1. Choose the correct alternative answer for each of the following questions.

   (1) Which number cannot represent a probability ?

   (A) $\dfrac{2}{3}$ (B) 1.5   (C) 15 %   D) 0.7

   (2) A die is rolled. What is the probability that the number appearing on upper face is less than 3 ?

   (A) $\dfrac{1}{6}$ (B) $\dfrac{1}{3}$ (C) $\dfrac{1}{2}$ D) 0

   (3) What is the probability of the event that a number chosen from 1 to 100 is a prime number ?

   (A) $\dfrac{1}{5}$ (B) $\dfrac{6}{25}$ (C) $\dfrac{1}{4}$ D) $\dfrac{13}{50}$

   (4) There are 40 cards in a bag. Each bears a number from 1 to 40. One card is drawn at random. What is the probability that the card bears a number which is a multiple of 5 ?

   (A) $\dfrac{1}{5}$ (B) $\dfrac{3}{5}$ (C) $\dfrac{4}{5}$ D) $\dfrac{1}{3}$

   (5) If $n(A) = 2$, $P(A) = \dfrac{1}{5}$,   then $n(S) = ?$

   (A) 10 (B) $\dfrac{5}{2}$ (C) $\dfrac{2}{5}$ D) $\dfrac{1}{3}$

2. Basketball players John, Vasim, Akash were practising the ball drop in the basket. The probabilities of success for John, Vasim and Akash are $\dfrac{4}{5}$, 0.83 and 58% respectively. Who had the greatest probability of success ?

3. In a hockey team there are 6 defenders , 4 offenders and 1 goalee. Out of these, one player is to be selected randomly as a captain. Find the probability of the selection that -

   (1) The goalee will be selected.   (2) A defender will be selected.

4. Joseph kept 26 cards in a cap, bearing one English alphabet on each card. One card is drawn at random. What is the probability that the card drawn is a vowel card ?

5. A balloon vendor has 2 red, 3 blue and 4 green balloons. He wants to choose one of them at random to give it to Pranali. What is the probability of the event that Pranali gets,

   (1) a red balloon   (2) a blue balloon   (3) a green balloon.

6. A box contains 5 red, 8 blue and 3 green pens. Rutuja wants to pick a pen at random. What is the probability that the pen is blue?

7. Six faces of a die are as shown below.

| A | B | C | D | E | A |
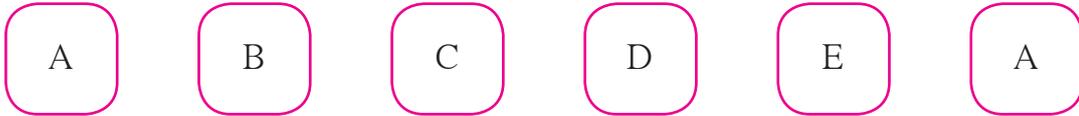
If the die is rolled once, find the probability of -

(1) 'A' appears on upper face.

(2) 'D' appears on upper face.

8. A box contains 30 tickets, bearing only one number from 1 to 30 on each. If one ticket is drawn at random, find the probability of an event that the ticket drawn bears   (1) an odd number   (2) a complete square number.

9. Length and breadth of a rectangular garden are 77 m and 50 m. There is a circular lake in the garden having diameter 14 m. Due to wind, a towel from a terrace on a nearby building fell into the garden. Then find the probability of the event that it fell in the lake.

77m
50m

10. In a game of chance, a spinning arrow comes to rest at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8.

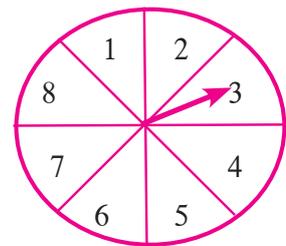All these are equally likely outcomes. Find the probability that it will rest at

(1) 8.

(2) an odd number.

(3) a number greater than 2.

(4) a number less than 9.

11. There are six cards in a box, each bearing a number from 0 to 5. Find the probability of each of the following events, that a card drawn shows,

(1) a natural number.

(2) a number less than 1.

(3) a whole number.

(4) a number is greater than 5.

12. A bag contains 3 red, 3 white and 3 green balls. One ball is taken out of the bag at random. What is the probability that the ball drawn is -

    (1) red.   (2) not red   (3) either red or white.

13. Each card bears one letter from the word 'mathematics' The cards are placed on a table upside down. Find the probability that a card drawn bears the letter 'm'.

14. Out of 200 students from a school, 135 like Kabbaddi and the remaining students do not like the game. If one student is selected at random from all the students, find the probability that the student selected dosen't like Kabbaddi.

15.★ A two digit number is to be formed from the digits 0, 1, 2, 3, 4. Repetition of the digits is allowed. Find the probability that the number so formed is a -

    (1) prime number   (2) multiple of 4

    (3) multiple of 11.

16.★ The faces of a die bear numbers 0, 1, 2, 3, 4, 5. If the die is rolled twice, then find the probability that the product of digits on the upper face is zero.

17. Do the following activity -

Activity I : Total number of students in your class, $n(S) = $ ☐

Number of students from your class, wearing spectacles, $n(A) = $ ☐

Probability of a randomly selected student wearing spectacles, $P(A) = $ ☐

Probability of a randomly selected student not wearing spectacles, $P(B) = $ ☐

Activity II : Decide the sample space yourself and fill in the following boxes.

Sample space

$\downarrow$

S = { ☐ }

$\downarrow$

n(S) = ☐

The condition for event A is 'getting an even number'.

$\downarrow$

A = { ☐ }

$\downarrow$

n(A) = ☐

$$\therefore \ P(A) = \frac{\boxed{}}{\boxed{}} = \boxed{}$$

N63IJ7

6   Statistics

- Measures of a central tendency-
  mean, median and mode from grouped frequency table.
- Graphical representation of statistical data -
  histogram, frequency polygon, pie diagram

Statistics is useful in many fields of life: for example, agriculture, economics, commerce, medicine, botany, biotechnology, physics, chemistry, education, sociology, administration etc. An experiment can have many outcomes. To assess the possibility of possible outcomes, one has to carry out the experiment on a large scale and keep the record meticulously. Possibilities of different outcomes can be assessed using the record. For this purpose, rules are formulated in statistics.

Francis Galton (1822-1911) has done much of fundamental work in statistics. He used to prepare questionnaires, distribute them among people and request them to fill them up. He collected information from a number of people and recorded their backgrounds, financial situations, likes and dislikes, health etc. on a large scale. By that time, it was known that the fingerprints of different people are different. He collected finger-prints of a large number of people and invented a method of their classification. Using statistical methods, he showed that the possibility of finger prints of two different people being identical is nearly zero. This result made it possible to identify a person from his finger-prints. This method of identifying criminals was accepted in the judiciary. He had done much work in the field of anthropology of humans and other animals also.

Francis Galton

Let's recall.

We usually find a specific property in the numerical data collected in a survey that the scores have a tendency to cluster around a particular score. This score is a representative number of the group. The number is called the measure of central tendency.

In the previous standards we have studied the measures of central tendency, namely the mean, median and mode, for ungrouped data.

Activity 1 : Measure the height in cm of all students in your class. We find that the heights of many students cluster near a specific number.

Activity 2 : Collect a number of fallen leaves of a peepal tree. Distribute the leaves among the students and ask them to measure the lengths of them. Record the lengths. We notice that their lengths tend to cluster around a number.

Now we are going to do some more study of the mean, median and mode. Let us know the symbols and the terminology required for it.

$$\text{The mean of statistical data} = \frac{\text{The sum of all scores}}{\text{Total no. of scores}} = \frac{\sum_{i=1}^{N} x_i}{N}$$

(Here x_i is the ith score)

Mean is denoted by $\overline{X}$ and it represents the average of the given data.

$$\overline{X} = \frac{\sum_{i=1}^{N} x_i}{N}$$

Let's learn.

Mean from classified frequency distribution

When the number of scores in a data is large, it becomes tedious to write all numbers in the above formula and take their sum. So we use some different methods to find the sum.

Sometimes, the large data collected from an experiment is presented in a table in the grouped form. In such a case, we cannot find the exact mean of statistical data. Hence, let us study a method which gives the approximate mean, or a number nearby.

Direct method

Let us study the method by an example.

Ex. : The following table shows the frequency distribution of the time required for each worker to complete a work . From the table find the mean time required to complete the job for a worker.

Time (Hrs.) for each to complete the work	15–19	20–24	25–29	30–34	35–39
No. of workers	10	15	12	8	5

Solution :

(1) Vertical columns are drawn as shown in the table.

(2) Classes are written in the first column.

(3) The class mark x_i is in the second column.

(4) In the third column, the number of workers, that is frequency (f_i) is written.

(5) In the fourth column, the product ($x_i \times f_i$) for each class is written.

(6) Then $\sum\limits_{i=1}^{N} x_i f_i$ is written.

(7) The mean is found using the formula

Class (Time-hours)	Class mark x_i	Frequency (No. of Workers) f_i	Class mark × Frequency $x_i f_i$
15–19	17	10	170
20–24	22	15	330
25–29	27	12	324
30–34	32	8	256
35–39	37	5	185
Total		$\sum f_i = 50$	$\sum x_i f_i = 1265$

$$\text{Mean} = \overline{X} = \frac{\sum x_i f_i}{N} = \frac{1265}{50} = 25.3 \quad \because \sum f_i = N$$

The mean time required to complete the work for a worker = 25.3 hrs. (Approx)

✐✐✐ Solved Examples ✐✐✐

Ex. (1) The percentage of marks of 50 students in a test is given in the following table. Find the mean of the percentage.

Percentage of marks	0–20	20–40	40–60	60–80	80–100
No. of students	3	7	15	20	5

Solution : The following table is prepared as per steps.

Class (Percentage of marks)	Class mark x_i	Frequency (No. of students) f_i	Class mark × frequency $x_i f_i$
0–20	10	3	30
20–40	30	7	210
40–60	50	15	750
60–80	70	20	1400
80–100	90	5	450
Total		$N = \sum f_i = 50$	$\sum x_i f_i = 2840$

$$\overline{X} = \frac{\sum x_i f_i}{\sum f_i}$$

$$= \frac{2840}{50}$$

$$= 56.8$$

∴ The mean of the percentage = 56.8

Ex. (2) The maximum temperatures in °C of 30 towns, in the last summer, is shown in the following table. Find the mean of the maximum temperatures.

Max. temp.	24–28	28–32	32–36	36–40	40–44
No. of towns	4	5	7	8	6

Solution :

Class (Temp. °C)	Class mark x_i	Frequency (No. of towns) f_i	Class mark × frequency $x_i f_i$
24–28	26	4	104
28–32	30	5	150
32–36	34	7	238
36–40	38	8	304
40–44	42	6	252
Total		$N = \sum f_i = 30$	$\sum x_i f_i = 1048$

$$\text{Mean} = \overline{X} = \frac{\sum x_i f_i}{\sum f_i} = \frac{1048}{30} = 34.9 \ °C$$

Assumed mean method

In the examples solved above, we see that some times the product $x_i f_i$ is large. Hence it becomes difficult to calculate the mean by direct method. So let us study another method, called the 'assumed mean method'. Finding the mean becomes simpler if we use addition and division in this method.

For example, we have to find the mean of the scores 40, 42, 43, 45, 47 and 48.

The obeservation of the scores reveals that the mean of the data is more than 40. So let us assume that the mean is 40. 40–40 = 0, 42 – 40 = 2, 43–40 = 3, 45–40 = 5, 47 – 40 = 7, 48 – 40 = 8 These are called 'deviations'. Let us find their mean. Adding this mean to the assumed mean, we get the mean of the data.

That is, mean = assumed mean + mean of the deviations

$$\overline{X} = 40 + \left(\frac{0+2+3+5+7+8}{6} \right) = 40 + \frac{25}{6} = 40 + 4\frac{1}{6} = 44\frac{1}{6}$$

Using the symbols-

A- for assumed mean; d- for deviation and $\overline{d}$ - for the mean of the deviations, the formula for mean of the given data can be briefly written as $\overline{X} = A + \overline{d}$.

Let us solve the same example taking 43 as assumed mean. For this, let us find the deviations by subtracting 43 from each score.

$40 - 43 = -3$, $42 - 43 = -1$, $43 - 43 = 0$, $45 - 43 = 2$, $47 - 43 = 4$, $48 - 43 = 5$

The sum of the deviations $= -3 -1 + 0 + 2 + 4 + 5 = 7$

Now, $\overline{X} = A + \overline{d}$

$= 43 + \left(\dfrac{7}{6}\right)$ (as the number of deviations is 6)

$= 43 + 1\dfrac{1}{6} = 44\dfrac{1}{6}$

Note that; use of assumed mean method reduces the work of calculations.

Also note that; taking any score, or any other convenient number as asssumed mean does not change the mean of the data.

Ex. : The daily sale of 100 vegetable vendors is given in the following table. Find the mean of the sale by assumed mean method.

Daily sale (Rupees)	1000-1500	1500-2000	2000-2500	2500-3000
No. of vendors	15	20	35	30

Solution : Assumed mean $= A = 2250$, $d_i = x_i - A$ is the deviation.

Class Daily sale (Rupees)	Class mark x_i	$d_i = x_i - A$ $= x_i - 2250$	Frequency (No. of vendors) f_i	Frequency $\times$ deviation $f_i d_i$
1000-1500	1250	-1000	15	-15000
1500-2000	1750	-500	20	-10000
2000-2500	2250→A	0	35	0
2500-3000	2750	500	30	15000
Total			$N = \sum f_i = 100$	$\sum f_i d_i = -10000$

The table is prepared according to the following steps :-

(1) Assumed mean, A is choosen as 2250. (Generally, the class mark of the class having maximum frequency is chosen as the assumed mean.)

(2) Classes of sale are written in the first column.

(3) Class marks are written in the second column.

(4) Values of $d_i = x_i - A = x_i - 2250$ are written in the third column.

(5) In the fourth column, the number of vendors and their sum is written as $\sum f_i$.

(6) In the fifth column, the product $(f_i \times d_i)$ and their sum is written as $\sum f_i d_i$.

$\overline{d}$ and $\overline{X}$ are calculated using the formulae.

$$\overline{d} = \frac{\sum f_i d_i}{\sum f_i} = -\frac{10000}{100} = -100 \qquad \therefore \ \text{mean } \overline{X} = A + \overline{d} = 2250 - 100 = 2150$$

The mean of sale is ₹ 2150.

Activity :- Solve the above example by direct method.

✿✿✿ Solved Examples ✿✿✿

Ex. (1) The following table shows the frequency table of daily wages of 50 workers in a trading company. Find the mean wages of a worker, by assumed mean method.

Daily Wages (Rs)	200–240	240–280	280–320	320–360	360–400
Frequency (No. of workers)	5	10	15	12	8

Solution : Let us take the assumed mean A = 300.

Class (₹ Wage)	Class mark x_i	$d_i = x_i - A$ $d_i = x_i - 300$	Frequency (No. of workers) f_i	Frequency × Deviation $f_i d_i$
200–240	220	−80	5	−400
240–280	260	−40	10	−400
280–320	300→A	0	15	0
320–360	340	40	12	480
360–400	380	80	8	640
Total			$\sum f_i = 50$	$\sum f_i d_i = 320$

$$\overline{d} = \frac{\Sigma f_i d_i}{\Sigma f_i} = \frac{320}{50} = 6.4$$

$$\text{Mean, } \overline{X} = A + \overline{d}$$

$$= 300 + 6.4$$

$$= 306.40$$

The mean of daily wages = 306.40 ₹

Step deviation method

We studied the direct method and assumed mean method to find the mean. Now we study one more method which reduces the calculations still further.

- Find the values of d_i as $d_i = x_i - A$ and write in the column.

- If we can find g, the G.C.D. of all d_i easily, we create a column for all u_i where $u_i = \dfrac{d_i}{g}$

- Find the mean $\overline{u}$ of all u_i.

- Using the formula $\overline{X} = A + \overline{u}\, g$, find the mean of the data.

Example : The amount invested in health insurance by 100 families is given in the following frequency table. Find the mean of investments using step deviation method.

Amount invested (₹)	800–1200	1200–1600	1600–2000	2000–2400	2400–2800	2800–3200
No. of families	3	15	20	25	30	7

Solution : Assumed mean A = 2200 observing all 'd_i's g = 400.

Class (Insurance ₹)	Class Mark x_i	$d_i = x_i - A$ $= x_i - 2200$	$u_i = \dfrac{d_i}{g}$	Frequency (No. of families) f_i	$f_i u_i$
800–1200	1000	−1200	−3	3	−9
1200–1600	1400	−800	−2	15	−30
1600–2000	1800	−400	−1	20	−20
2000–2400	2200→A	0	0	25	0
2400–2800	2600	400	1	30	30
2800–3200	3000	800	2	7	14
Total				$\sum f_i = 100$	$\sum f_i u_i = -15$

The above table is made using the following steps.

(1) The classes of investment are written in the first column.

(2) The values of x_i are written in the second column.

(3) The values of $d_i = x_i - A$ are written in the third column.

(4) The G.C.D of all values of d_i is 400. Therefore $g = 400$.

(5) The corresponding frequencies are written in the fifth column.

(6) The product $f_i \times u_i$ for each class is written in the sixth column.

The mean of u_i is found by the following formula.

$$\overline{u} = \frac{\sum f_i u_i}{\sum f_i} = \frac{-15}{100} = -0.15$$

$\overline{X} = A + \overline{u}\, g$

$= 2200 + (-0.15)\,(400)$

$= 2200 + (-60.00)$

$= 2200 - 60 = 2140$

∴ The mean of investments in health insurance = ₹ 2140.

Activity : Solve the above example by direct method and by assumed mean method and see that the mean found by any method is the same.

<p align="center">♨♨♨ **Solved Example** ♨♨♨</p>

Ex. (1) The following table shows the funds collected by 50 students for flood affected people. Find the mean of the funds.

Fund (Rupees)	0-500	500-1000	1000-1500	1500-2000	2000-2500	2500-3000
No. of students	2	4	24	18	1	1

If the number of scores in two consecutive classes is very low, it is convenient to club them. So, in the above example, we club the classes 0 – 500, 500 – 1000 and 2000 – 2500, 2500 – 3000. Now the new table is as follows

Fund (Rupees)	0-1000	1000-1500	1500-2000	2000-3000
No. of students	6	24	18	2

Solution : Let A = 1250, examining all d_i, g = 250.

Class Fund (₹)	Class mark x_i	$d_i = x_i - A$ $= x_i - 1250$	$u_i = \dfrac{d_i}{g}$	Frequency f_i	$f_i u_i$
0-1000	500	-750	-3	6	-18
1000-1500	1250➙A	0	0	24	0
1500 – 2000	1750	500	2	18	36
2000-3000	2500	1250	5	2	10
Total				$\sum f_i$ = 50	$\sum f_i u_i$ = 28

$$\overline{u} = \frac{\sum f_i u_i}{\sum f_i} = \frac{28}{50} = 0.56,$$

$$\overline{u} \, g = 0.56 \times 250 = 140$$

$$\overline{X} = A + g\,\overline{u} = \quad 1250 + 140 = 1390$$

∴ the average of the funds is ₹ 1390.

Activity -

1. Solve the above example by direct method.

2. Verify that the mean calculated by assumed mean method is the same.

3. Find the mean in the above example by taking A = 1750.

1. The following table shows the number of students and the time they utilized daily for their studies. Find the mean time spent by students for their studies by direct method.

Time (hrs.)	0-2	2-4	4-6	6-8	8-10
No. of students	7	18	12	10	3

2. In the following table, the toll paid by drivers and the number of vehicles is shown. Find the mean of the toll by 'assumed mean' method.

Toll (Rupees)	300-400	400-500	500-600	600-700	700-800
No. of vehicles	80	110	120	70	40

3. A milk centre sold milk to 50 customers. The table below gives the number of customers and the milk they purchased. Find the mean of the milk sold by direct method.

Milk Sold (Litre)	1-2	2-3	3-4	4-5	5-6
No. of Customers	17	13	10	7	3

4. A frequency distribution table for the production of oranges of some farm owners is given below. Find the mean production of oranges by 'assumed mean' method.

Production (Thousand rupees)	25-30	30-35	35-40	40-45	45-50
No. of farm owners	20	25	15	10	10

5. A frequency distribution of funds collected by 120 workers in a company for the drought affected people are given in the following table. Find the mean of the funds by 'step deviation' method.

Fund (Rupees)	0-500	500-1000	1000-1500	1500-2000	2000-2500
No. of workers	35	28	32	15	10

6. The following table gives the information of frequency distribution of weekly wages of 150 workers of a company. Find the mean of the weekly wages by 'step deviation' method.

Weekly wages (Rupees)	1000-2000	2000-3000	3000-4000	4000-5000
No. of workers.	25	45	50	30

There was a science exhibition in a city for two days. A school sent two boys and two girls to participate in the exhibition. There were ten hotels, within a distance of one kilometer, from the venue of exhibition. Their rates of meals, in the ascending order were rupees 40, 45, 60, 65, 70, 80, 90, 100 and 500. They had to choose one of them for the dinner.

The average of rates in all the hotels was ₹ $\frac{1130}{10}$ = 113.

Which hotel do you think they chose? Except the rate ₹ 500, all others were less than ₹ 113. The students decided to choose a hotel having medium rate. The first day they chose the hotel with rate ₹ 70 and on the next day, the hotel with the rate ₹ 80/-.

This example shows that sometimes the median is used instead of the mean.

In the previous standard we have studied the concept of a median.

- If the numbers in a data are arranged in the ascending order, the number at the middle position is called the median of the data.

- The median divides the array of numbers in two equal parts, that is the number of scores below and above the median is equal.

- The scores are written as $k_1 \leq k_2 \leq k_3 \ldots \ldots \leq k_n$.

- If the number of scores is odd, then the $\frac{n+1}{2}$ th score is the median of the data. That is, the number of scores below as well as above $k_{\frac{n+1}{2}}$ is $\frac{n-1}{2}$; verify the fact by taking n = 2m + 1.

- If the number of the scores is even, then the mean of the middle two terms is the median. This is because the number of terms below $k_{\frac{n}{2}}$ and above $k_{\frac{n+2}{2}}$ is equal, which is $\frac{n-2}{2}$. Verify this by taking n = 2m.

- Hence the mean of $\frac{n}{2}$ th and $\frac{n+2}{2}$ th term is the median of the data.

Ex. (1) In 32, 33, 38, 40, 43, 48, 50; the fourth number is at the middle. Hence the median of the data is 40

Ex. (2) In 61, 62, 65, 66, 68, 70, 74, 75 ; the number of scores is 8, that is even. Therefore, the fourth and the fifth numbers are at the middle, which are 66 and 68. Hence the median = $\frac{66+68}{2}$ = 67

Median for grouped frequency distribution

When the number of scores in a data is large, it is difficult to arrange them in ascending order. In such case, the data is divided into groups. So let us study, with an example, how the median of grouped data is found.

Ex. The scores 6, 8, 10.4, 11, 15.5, 12, 18 are grouped in the following table.

Class	Tally Marks	Frequency
6-10	II	2
11-15	II	2
16-20	I	1

Class	Tally Marks	Frequency
5.5-10.5	III	3
10.5-15.5	II	2
15.5-20.5	II	2

We could not record the scores 10.4 and 15.5 in the first table, as they cannot be placed in any of the classes 6-10, 11-15, 16-20. We know that in such a case the classes are made continuous.

For this, in the first table, the lower class limits are reduced by 0.5 and the upper class limits are increased by 0.5 and the second table is prepared. In the second table, the score 15.5 is placed in the class 15.5-20.5.

Note that if the method of making groups is changed, the frequency distribution may change.

Let's remember!

In the above table, the class mark of 6-10 is $= \dfrac{6+10}{2} = \dfrac{16}{2} = 8$;

Similarly, the class mark of 5.5-10.5 is $= \dfrac{5.5+10.5}{2} = \dfrac{16}{2} = 8$.

This shows that, if the classes are made continuous, the class marks do not change

Solved Example :

The following table shows frequency distribution of marks of 100 students of 10th class which they obtained in a practice examination. Find the median of the marks.

Marks in exam	0-20	20-40	40-60	60-80	80-100
No. of students	4	20	30	40	6

Solution :   N = 100

$\therefore$ $\dfrac{N}{2}$ = 50. Hence the 50th number will be the approximate median. Hence we have to find out the class which contains the 50th term. Writing the cumulative frequencies less than the upper limit, we can find it.

So, let us prepare less than cumulative frequency distribution table.

Class (Student's marks)	No. of students f_i	Cumulaive frequency less than the upper limit cf
0-20	4	4
20-40	20	24
40-60	30	54
60-80	40	94
80-100	6	100

- From the table, the 50th score is in the class 40-60. The class which contains the median, is called the **median class**. So, here 40-60 is the median class.

- The lower class limit of 40-60 is 40. Its frequency is 30.

- Out of the first 50 scores, 24 scores are less than 40. The remaining 50 - 24 = 26 are in class (40-60). The 50th score in that class is estimated as follows.

- 26 out of 30 scores in the class 40-60, are upto the 50th score and the class interval is 20. So it is assumed that, the 50th score is more than 40 by $\dfrac{26}{30} \times 20$.

$\therefore$ it is approximately 40 + $\dfrac{26}{30}$ $\times$ 20 = 40 + $\dfrac{52}{3}$ = $57\dfrac{1}{3}$.

$\therefore$ median = $57\dfrac{1}{3}$

- We can formulate this as follows,

$$\text{Median} = L + \left[\dfrac{\dfrac{N}{2} - cf}{f}\right] \times h$$

In the formula,

L = Lower class limit of the median class,

N = Sum of frequencies

h = Class interval of the median class,

f = Frequency of the median class

cf = Cumulative frequency of the class preceding the median class.

In the above example; $\dfrac{N}{2}$ = 50, cf = 24, h = 20, f = 30, L = 40,

$$\text{Median} = L + \left[\dfrac{\dfrac{N}{2} - cf}{f}\right] \times h \quad \ldots \ldots \text{(Formula)}$$

$$= 40 + \left(\dfrac{50 - 24}{30}\right) \times 20$$

$$= 40 + \dfrac{26 \times 20}{30}$$

$$= 40 + 17\dfrac{1}{3}$$

$$= 57\dfrac{1}{3}$$

Let's remember!

✦ If the given classes are not continuous, we have to make them continuous to find out the median.

✦ It is difficult to write the scores in the asscending order when the number of scores is large. So the data is classified into groups. It is not possible to find the exact median of a classified data, but the approximate median is found by the formula.

$$\text{Median} = L + \left[\dfrac{\dfrac{N}{2} - cf}{f}\right] \times h$$

Ex. (1) Observe the following frequency distribution table. It shows the distances travelled by 60 public transport buses in a day. Find the median of the distance travelled.

Daily distance travelled (in Km)	200–209	210–219	220–229	230–239	240–249
No. of buses	4	14	26	10	6

Solution : (1) The classes in the table are not continuous.

The upper class limit of a class and the lower class of its succeeding class differ by 1.

∴ Let us subtract 1 ÷ 2 = 0.5 from the lower class limit of each class and add to the upper class limit of each class, and make the classes continuous.

(2) Make a column of cumulative frequency 'less than' in the new table showing the continuous classes.

Given Class	Continuous classes	Frequency f_i	Cumulative frequency less than
200–209	199.5–209.5	4	4
210–219	209.5–219.5	14	18 → cf
220–229	219.5–229.5	26 → f	44
230–239	229.5–239.5	10	54
240–249	239.5–249.5	6	60

Here, total of frequencies = Σf_i = N = 60 ∴ $\dfrac{N}{2}$ = 30.

∴ 30th score is the approximate median.

First 18 scores are less than 219.5 and the remaining, 30 − 18 = 12 scores are in the class 219.5 − 229.5. Therefore, 219.5 − 229.5 is the median class. The cumulative frequency of the class 219.5−229.5 is 44.

In the formula,

L = Lower class limit = 219.5, h = Class interval of the median class = 10

cf = The frequency of the class preceding the median class = 18,

f = The frequency of the median class = 26

$$\text{Median} = L + \left[\dfrac{\dfrac{N}{2} - cf}{f} \right] \times h$$

$\therefore$ Median = $219.5 + \left(\dfrac{30-18}{26}\right) \times 10$

$= 219.5 + \left(\dfrac{12 \times 10}{26}\right)$

$= 219.50 + 4.62$

$= 224.12$

$\therefore$ The median of the distance travelled is = 224.12 Km

Ex. (2) The following table shows the ages of persons who visited a museum on a certain day. Find the median age of the persons visiting the museum.

Age (Years)	No. of persons
Less than 10	3
Less than 20	10
Less than 30	22
Less than 40	40
Less than 50	54
Less than 60	71

Solution : The given cumulative frequency table is of the 'less than' form. So, we will have to decide the true class limits first. We know that, the 'less than' cumulative frequency is associated with the upper class limits. The upper class limit of the first class is 10. The age of any person is a positive number, so the first class must be 0-10. The upper class limit of the next class is 20, so the second class must be 10-20. In this way, make the classes of interval 10. In this way the last class is 50-60. So the given table can now be rewritten as follows.

Age (years)	Class	No. of persons (Frequency)	Cumulative frequency Less than
Less than 10	0-10	3	3
Less than 20	10-20	10 – 3 = 7	10
Less than 30	20-30	22 – 10 = 12	22 → cf
Less than 40	30-40	40 – 22 = 18 → f	40
Less than 50	40-50	54 – 40 = 14	54
Less than 60	50-60	71 – 54 = 17	71

Here N = 71 $\therefore$ $\dfrac{N}{2}$ = 35.5 and h = 10

The number 35.5 is in the class 30-40, hence it is the median class. The cumulative frequency of its precending class is 22, $\therefore$ cf = 22, L = 30, f = 18.

$$\text{Median} = L + \left[\dfrac{\dfrac{N}{2} - cf}{f}\right] \times h$$

$$= 30 + (35.5-22)\,\dfrac{10}{18}$$

$$= 30 + (13.5)\,\dfrac{10}{18}$$

$$= 30 + 7.5$$

$$= 37.5$$

$\therefore$ the median age of the persons visiting the museum is = 37.5 years

Practice Set 6.2

1. The following table shows classification of number of workers and the number of hours they work in a software company. Find the median of the number of hours they work.

Daily No. of hours	8-10	10-12	12-14	14-16
Number of workers	150	500	300	50

2. The frequency distribution table shows the number of mango trees in a grove and their yield of mangoes. Find the median of data.

No. of Mangoes	50-100	100-150	150-200	200-250	250-300
No. of trees	33	30	90	80	17

3. The following table shows the classification of number of vehicles and their speeds on Mumbai-Pune express way. Find the median of the data.

Average Speed of Vehicles(Km/hr)	60-64	64-69	70-74	75-79	79-84	84-89
No. of vehicles	10	34	55	85	10	6

4. The production of electric bulbs in different factories is shown in the following table. Find the median of the productions.

No. of bulbs produced (Thousands)	30−40	40−50	50−60	60−70	70−80	80−90	90−100
No. of factories	12	35	20	15	8	7	8

Let's learn.

Mode for grouped frequency distribution

We know that the score repeating maximum number of times in a data is called the mode of the data.

For example, a company manufactures bicycles of different colours. To know which colour is most wanted, the company needs to know the mode. If a company manufactures many items, it may want to know which item sells most. In such cases, the mode is needed.

We have learnt the method of finding the mode of an ungrouped data.

Now let us study the method of estimation of mode of grouped data.

The following formula is used for the purpose.

$$\text{Mode} = L + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right] \times h$$

In the above formula,

L = Lower class limit of the modal class.

f_1 = Frequency of the modal class.

f_0 = Frequency of the class preceding the modal class.

f_2 = Frequency of the class succeeding the modal class.

h = Class interval of the modal class.

Let us see, with an example, how the mode is estimated using the above formula.

Ex.(1) The classification of children according to their ages, playing on a ground is shown in the following table. Find the mode of ages of the children.

Age-group of children (Yrs)	6-8	8-10	10-12	12-14	14-16
No. of children	43	58 →f_0	70 →f_1	42 →f_2	27

From the table, we note that the maximum number of children is of the age-group 10-12. So the modal class is 10-12.

Solution : Here f_1 = 70, and modal class is 10-12.

∴ in the given example,

L = Lower class limit of the modal class = 10

h = Class interval of the modal class = 2

f_1 = Frequency of the modal class = 70

f_0 = Frequency of the class preceding the modal class = 58

f_2 = Frequency of the class succeeding the modal class = 42

$$\text{Mode} = L + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$$

$$= 10 + \left[\frac{70 - 58}{2(70) - 58 - 42}\right] \times 2$$

$$= 10 + \left[\frac{12}{140 - 100}\right] \times 2$$

$$= 10 + \left[\frac{12}{40}\right] \times 2$$

$$= 10 + \frac{24}{40}$$

$$= 10 + 0.6$$

$$= 10.6$$

∴ the mode of the ages of children playing on the ground is 10.6 Years.

Ex. (2) The following frequency distribution table shows the classification of the number of vehicles and the volume of petrol filled in them. Find the mode of the volume.

Petrol filled (Litre)	1−3	4−6	7−9	10−12	13−15
No. of vehicle	33	40	27	18	12

Solution : The given classes are not continuous. So, let us make them continuous and rewrite the table.

Class	Continuous classes	Frequency
1−3	0.5−3.5	$33 \rightarrow f_0$
4−6	3.5−6.5	$40 \rightarrow f_1$
7−9	6.5−9.5	$27 \rightarrow f_2$
10−12	9.5−12.5	18
13−15	12.5−15.5	12

From the above table, the modal class is 3.5−6.5

$$\text{Mode} = L + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$$

$$\text{Mode} = 3.5 + \left[\frac{40 - 33}{2(40) - 33 - 27}\right] \times h$$

$$= 3.5 + \left[\frac{7}{80 - 60}\right] \times 3$$

$$= 3.5 + \frac{21}{20}$$

$$= 3.5 + 1.05$$

$$= 4.55$$

∴ The mode of the volume of petrol filled is = 4.55 litre.

1. The following table shows the information regarding the milk collected from farmers on a milk collection centre and the content of fat in the milk, measured by a lactometer. Find the mode of fat content.

Content of fat (%)	2-3	3-4	4-5	5-6	6-7
Milk collected (Litre)	30	70	80	60	20

2. Electricity used by some families is shown in the following table. Find the mode for use of electricity.

Use of electricity (Unit)	0-20	20-40	40-60	60-80	80-100	100-120
No. of families	13	50	70	100	80	17

3. Grouped frequency distribution of supply of milk to hotels and the number of hotels is given in the following table. Find the mode of the supply of milk.

Milk (Litre)	1-3	3-5	5-7	7-9	9-11	11-13
No. of hotels	7	5	15	20	35	18

4. The following frequency distribution table gives the ages of 200 patients treated in a hospital in a week. Find the mode of ages of the patients.

Age (years)	Less than 5	5-9	10-14	15-19	20-24	25-29
No. of patients	38	32	50	36	24	20

Activity :-

- Find the mean weight of 20 students in your class.
- Find the mode of sizes of shirts of students in your class
- Every student in your class should measure his/her own pulse rate, note the pulse rates of all students and find the mode of the pulse rate.
- Measure the height of every student in the class, prepare a grouped frequency distribution table and find the median of the heights.

We have studied the central tendencies mean, median and mode. Before selecting any of these measures, we have to know the purpose of its selection clearly.

Suppose, we have to judge the performance of five divisions of standard 10 in the internal examination. For the purpose, we have to find the 'mean' of marks of students in each division.

If we have to make two groups of students in a division based on their marks in the examination, we have to find the 'median' of their marks

If a 'bachat' group producing chalks wants to know about the colour of chalks having maximum demand, it will have to choose the 'mode'.

Pictorial representation of statistical data

The mean, median or mode of a numerical data or analysis of the data is useful to draw some useful inferences.

We know that tabulation is one of the methods of representing numerical data in brief. But a table does not quickly reveal some aspects of the data. A common man is interested in the important aspects of a data. For example, annual budget, information about a game, etc. Let us think of another way of data representation for the purpose.
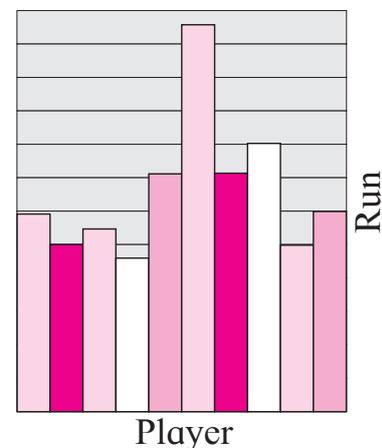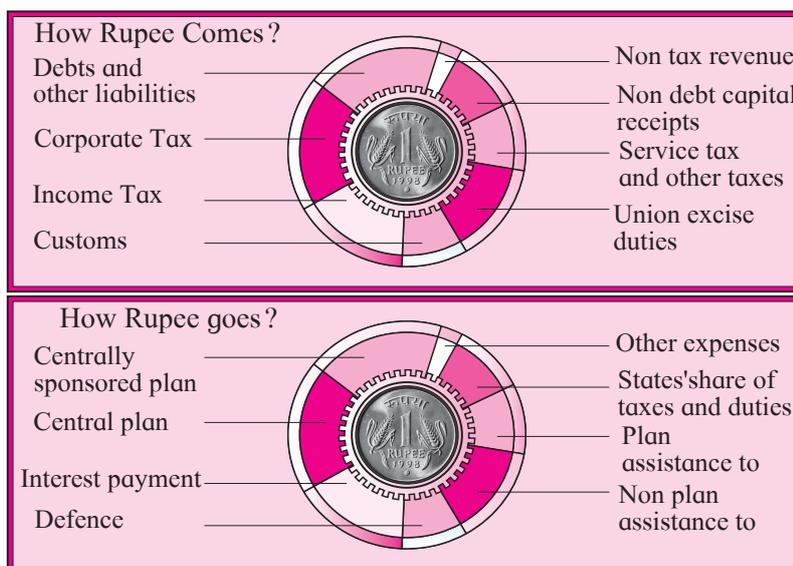
Pictorial and graphical presentation are attractive methods of data interpretation. The tree chart below shows different methods of data interpretation.

```
                          ┌──────────────────┐
                          │ Data Presentation│
                          └────────┬─────────┘
          ┌────────────────────────┼────────────────────────┐
   ┌───────────┐          ┌──────────────────────┐    ┌─────────────────┐
   │ Tabulation│          │ Pictorial Presentation│    │ Graphical Method│
   └─────┬─────┘          └──────────┬───────────┘    └────────┬────────┘
     ┌───┴────┐              ┌────────┴────────┐          ┌─────┴──────┐
┌─────────┐┌────────┐  ┌───────────┐  ┌───────────┐  ┌────────────┐
│Ungrouped││Grouped │  │Bar Diagram│  │Pie Diagram│  │ Line  Graph│
└─────────┘└────────┘  └─────┬─────┘  └───────────┘  └────────────┘
   ┌───────────┬────────────┼────────────┐        ┌─────────┬──────────┐
┌──────┐  ┌───────┐  ┌─────────┐  ┌──────────┐ ┌──────────┐┌──────────┐
│Simple│  │ Joint │  │ Divided │  │Percentage│ │Histogram ││Frequency │
└──────┘  └───────┘  └─────────┘  └──────────┘ └──────────┘│ polygon  │
                                                           └──────────┘
```
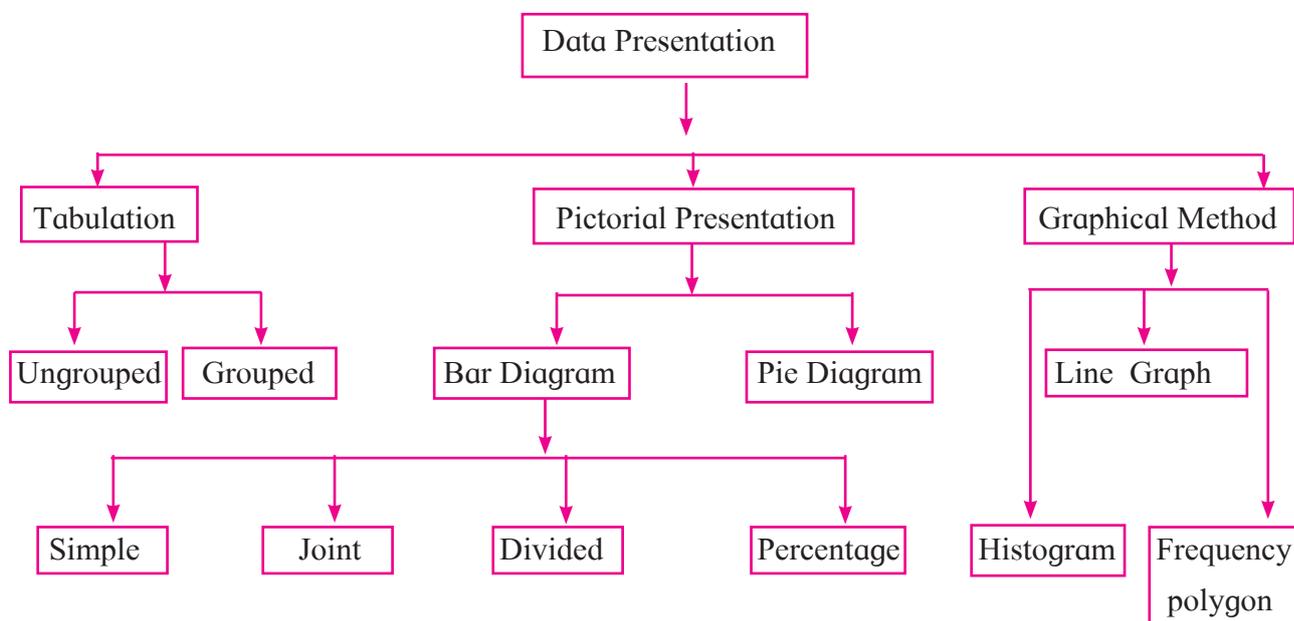
We have studied some of these methods and graphs in previous standards. Now we will learn a histogram, a frequency polygon and a pie diagram.

Florence Nightingale (1820-1910) The lady is considered as an idol in the field of nursing. She was devoted to the work of caring for the wounded and the sick. In the crimean war, she nursed wounded soldiers and saved their lives. She is also known for her fundamental work is the field of statistics. She kept a systematic record of the conditions of wounded soldiers, treatments given to them and the results of the treatments and deduced important conclusions. The cause of the death of soldiers was more often a disease like typhoid or cholera and not the wounds in the war. The causes of the diseases were lack of cleanliness of the surronding, polluted water and crowded dwelling of the patients. Florence exhibited the information in the form of graphs, and pie charts to convince the people. She showed that proper treatments and observing the rules of cleanliness decreases the death rate considerably. The municipalities accepted her observations, that to maintain the hygiene of town, good drainage system and clean drinking water for everyone are necessary. Her work established that systematic records and the statistical methods are useful in drawing reliable inferences.

**Histogram** Study the following example to know about a histogram and how to draw it.

Ex : The table below shows the net asset value (NAV) per unit of mutual funds of some companies.

Draw a histogram representing the information.

NAV (₹)	8−9	10−11	12−13	14−15	16−17
No. of mutual funds	20	40	30	25	15

Solution : The given classes are not continuous. Lets make the classes continuous.

Continuous Classes	7.5−9.5	9.5−11.5	11.5−13.5	13.5−15.5	15.5−17.5
Frequency	20	40	30	25	15
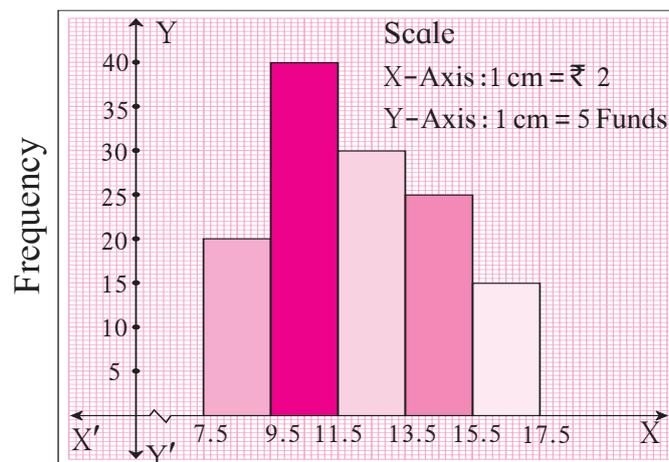
fig 6.1

Method of drawing a histogram :

1. If the given classes are not continuous, make them continuous. Such classes are called extended class intervals.

2. Show the classes on the X- axis with a proper scale.

3. Show the frequencies of the Y- axis with a proper scale.

4. Taking each class as the base, draw rectangles with heights proportional to the frequencies.

Note :

On the X-axis, a mark '⎯⋁⎯' is called the krink mark and it is shown between the origin and the first class. It means, there are no observations upto the first class. The mark can be used on the Y- axis also, if needed. This enables us to draw a graph of optimum size.

<div align="center">

Practice Set 6.4

</div>

1. Draw a histogram of the following data.

Height of student (cm)	135−140	140−145	145−150	150−155
No. of students	4	12	16	8

2. The table below shows the yield of jowar per acre. Show the data by histogram.

Yield per acre (quintal)	2−3	4−5	6−7	8−9	10−11
No. of farmers	30	50	55	40	20

3. In the following table, the investment made by 210 families is shown. Present it in the form of a histogram.

Investment (Thousand Rupees)	10−15	15−20	20−25	25−30	30−35
No. of families	30	50	60	55	15

4. Time alloted for the preparation of an examination by some students is shown in the table. Draw a histogram to show the information.

Time (minutes)	60−80	80−100	100−120	120−140	140−160
No. of students	14	20	24	22	16

<div align="center">

Let's learn.

</div>

Frequency polygon

The information in a frequency table can be presented in various ways. We have studied a histogram. A frequency polygon is another way of presentation.

Let us study two methods of drawing a frequency polygon.

(1) With the help of a histogram   (2) Without the help of a histogram.

(1) We shall use the histogram in figure 6.1 to learn the method of drawing a frequency polygon.
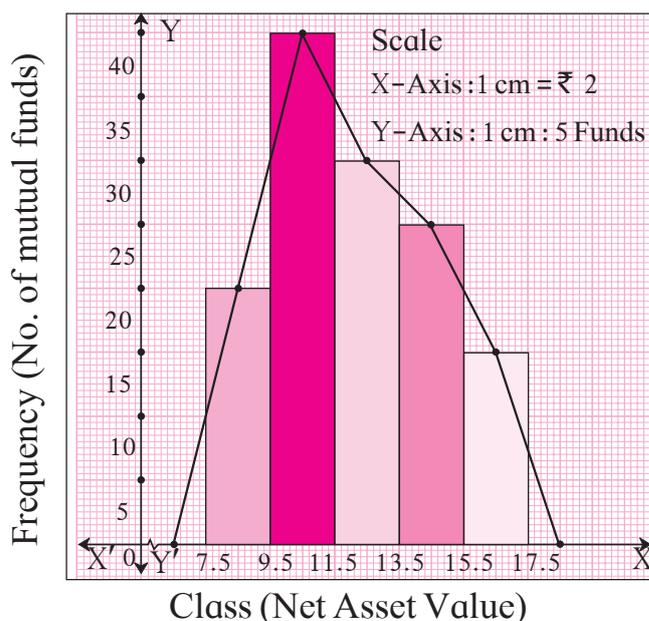
Fig. 6.2

1. Mark the mid-point of upper side of each rectangle in the histogram.

2. Assume that a rectangle of zero height exists preceeding the first rectangle and mark its mid-point. Similarly, assume a rectangle succeeding the last rectangle and mark its mid-point.

3. Join all mid-points in order by line segments.

   The closed figure so obtained is the frequency polygon.

(2) Observe the following table. It shows how the coordinates of points are decided to draw a frequency polygon, without drawing a histogram.

Class	Continuous class	Class mark	Frequency	Coordinates of points
6 – 7	5.5 – 7.5	6.5	0	(6.5, 0)
8 – 9	7.5 – 9.5	8.5	20	(8.5, 20)
10 – 11	9.5 – 11.5	10.5	40	(10.5, 40)
12 – 13	11.5 – 13.5	12.5	30	(12.5, 30)
14 – 15	13.5 – 15.5	14.5	25	(14.5, 25)
16 – 17	15.5 – 17.5	16.5	15	(16.5, 15)
18 – 19	17.5 – 19.5	18.5	0	(18.5, 0)

The points corresponding to the coordinates in the fifth column are plotted. Joining them in order by line segments, we get a frequency polygon. The polygon is shown in figure 6.3. Observe it.

Scale
X-Axis : 1 cm = 2 rupees
Y-Axis : 1 cm = 5 funds

Class (Net Asset Value)
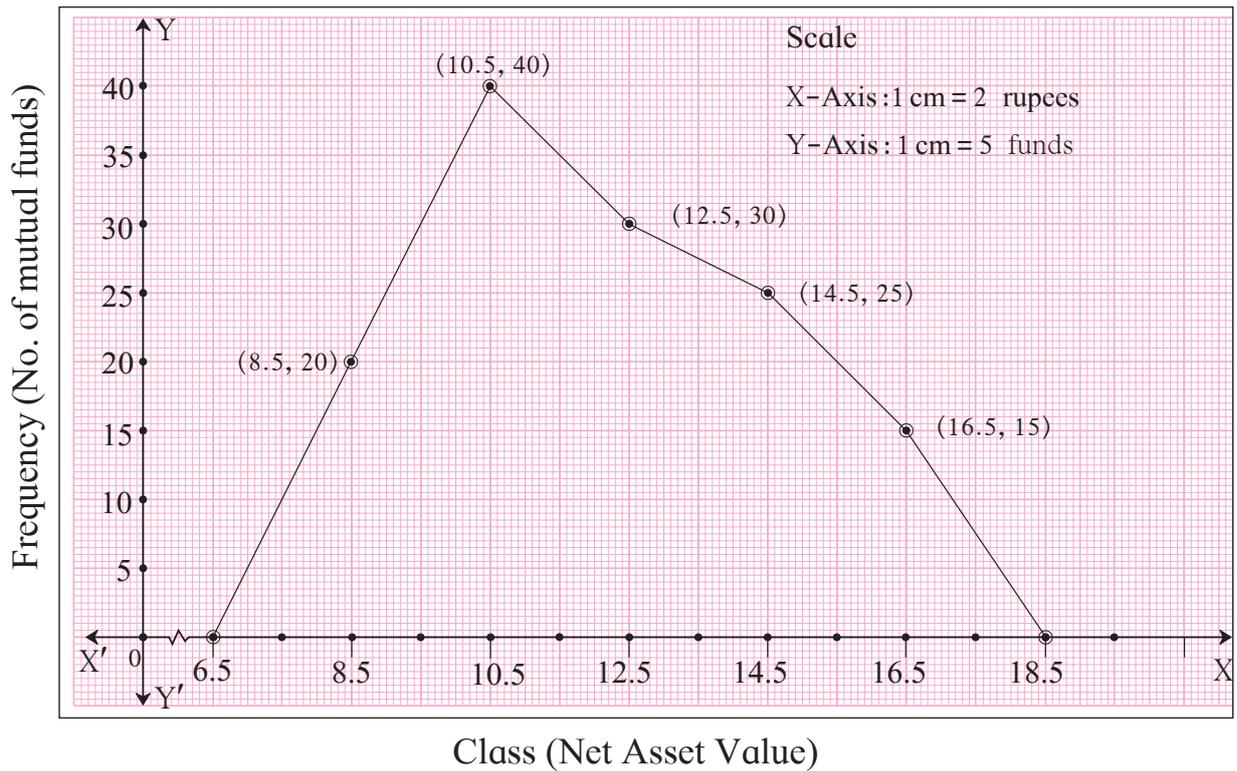
fig 6.3

❃❃❃ **Solved Examples** ❃❃❃

Ex. (1) Answer the following questions based on the frequency polygon given in the adjacent figure.

(1) Write frequency of the class 50–60.

(2) State the class whose frequency is 14.

(3) State the class whose class mark is 55.

(4) Write the class in which the frequency is maximum.

(5) Write the classes whose frequencies are zero.

Scale
X-Axis : 1 cm = 2 rupees
Y-Axis : 1 cm : 5 funds
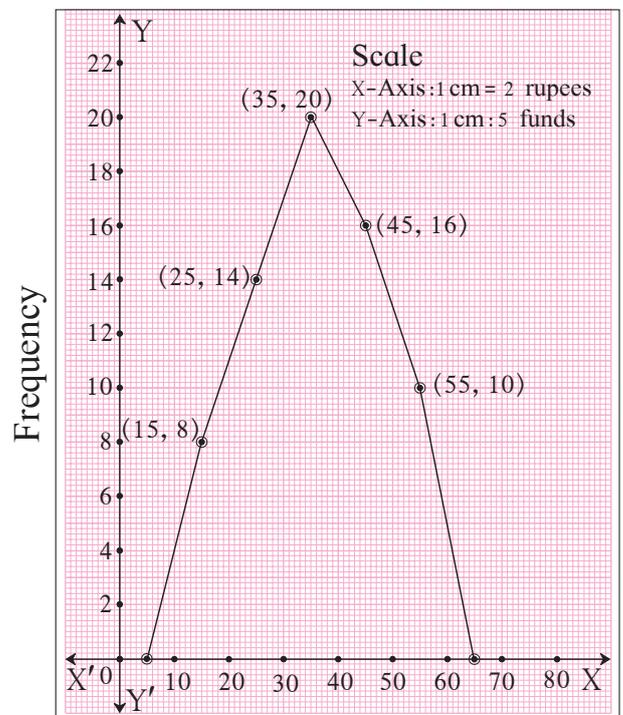
Class

fig 6.4

Solution :

(1) The class marks are on the X- axis. The point whose x- coordinate is 55 (as the mid-point of the class 50−60 is 55.) y-coordinate is 10. So, the frequency of the class 50−60 is 10.

(2) The frequencies are shown on the Y-axis. The x-coordinate of the point whose y-coordinate is 14, is 25. Note the mark 14 on the Y-axis . The class mark of the class 20−30 is 25. Hence, the frequency of the class 20−30 is 14.

(3) The class mark of the class 50−60 is 55.

(4) The frequency is shown on the Y-axis. On the polygon the maximum value of the y- coordinate is 20. Its corresponding x- coordinate is 35, which is the mark of the class 30−40. Therefore, the maximum frequency is in the class 30−40.

(5) The frequencies of the classes 0−10 and 60−70 are zero.

Ex. (2) The following table shows the weights of children and the number of children. Draw a frequency polygon showing the information.

Weight of children (kg)	18−19	19−20	20−21	21−22	22−23	23−24
No. of children	4	13	15	19	17	6

Let us prepare a table showing the co-ordinates necessary to draw a frequency polygon.

Class	18−19	19−20	20−21	21−22	22−23	23−24
Class mark	18.5	19.5	20.5	21.5	22.5	23.5
Frequency	4	13	15	19	17	6
Coordinates of points	(18.5, 4)	(19.5,13)	(20.5,15)	(21.5,19)	(22.5,17)	(23.5,6)

Scale
X−Axis : 2 cm = 1 kg
Y−Axis : 1 cm = 4 children

Fig. 6.5

Class mark (Weight-kg)
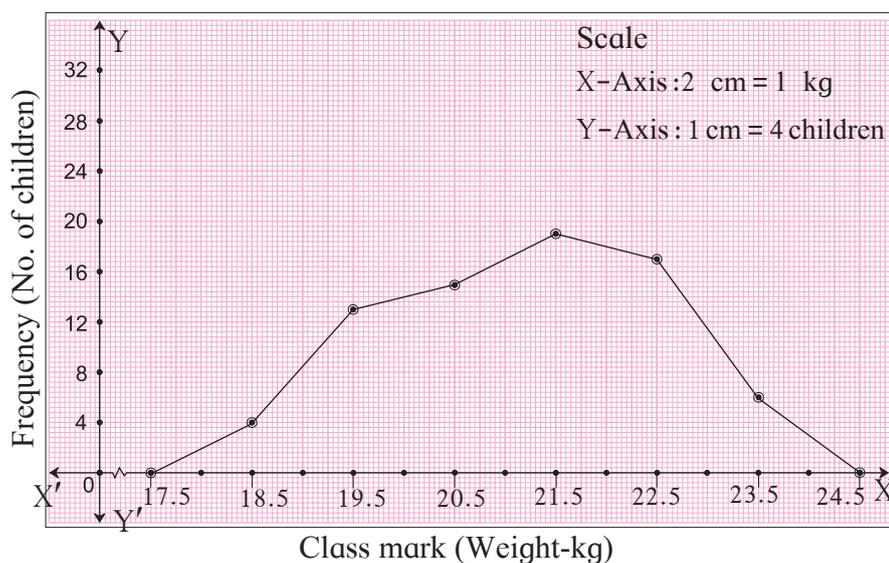
1. Observe the following frequency polygon and write the answers of the questions below it.

fig 6.6
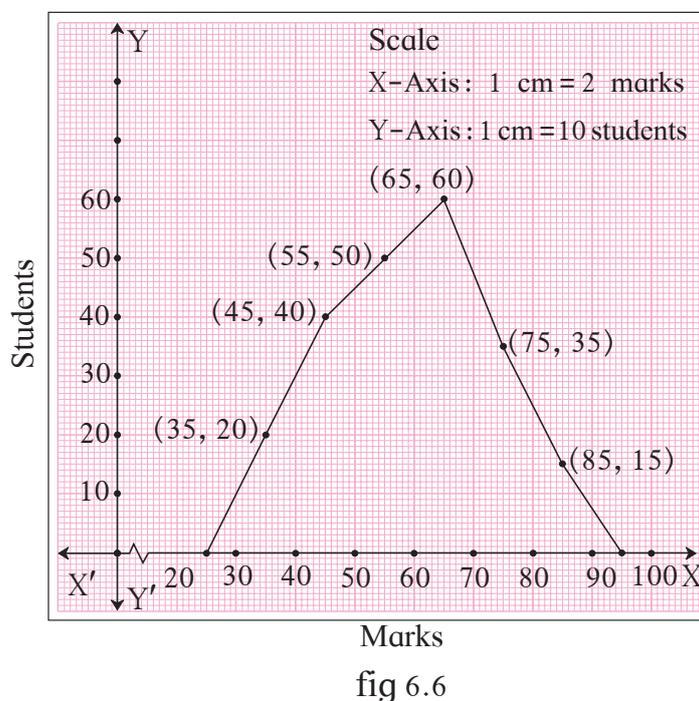
(1) Which class has the maximum number of students?

(2) Write the classes having zero frequency.

(3) What is the class-mark of the class, having frequency of 50 students?

(4) Write the lower and upper class limits of the class whose class mark is 85.

(5) How many students are in the class 80-90?

2. Show the following data by a frequency polygon.

Electricity bill (₹)	0-200	200-400	400-600	600-800	800-1000
Families	240	300	450	350	160

3. The following table shows the classification of percentages of marks of students and the number of students. Draw a frequency polygon from the table.

Result (Percentage)	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of students	7	33	45	65	47	18	5

Pie diagram

In the previous standards, we have seen the following figures in Geography and Science. Such graphs are called pie diagrams.

Land
29%

71%
Water
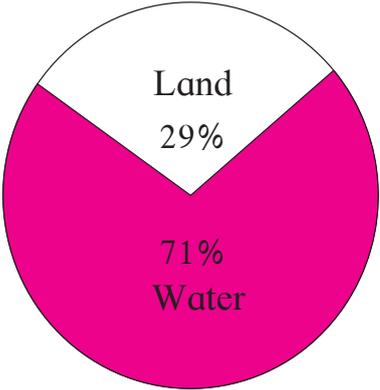
Proportion of land and water on the earth

Oxygen
21%

Other gases
1%

78%
Nitrogen
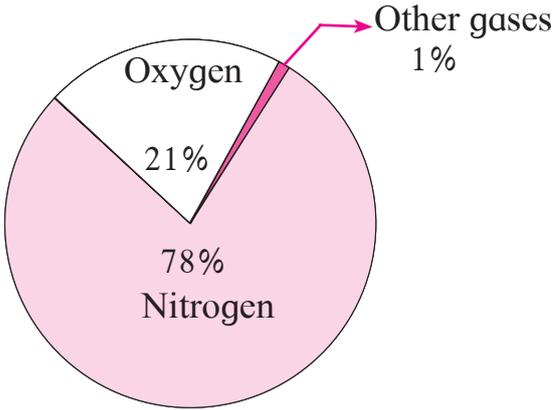
Proportion of constituents of air

fig 6.7

In a pie diagram, the numerical data is shown in a circle. Different components of a data are shown by proportional sectors of the circle.

In figure 6.8, seg OA and seg OB are radii of a circle with centre O.

$\angle$ AOB is the central angle.

The shaded region O – AXB is a sector of the circle.
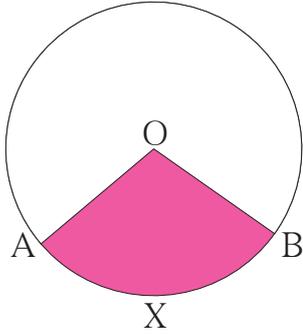
fig 6.8

Let's learn.

Reading of Pie diagram

The following example illustrates how a pie chart gives information at a glance.

120 students of standard 10 were asked which game they like. The information obtained is shown in the adjacent pie diagram. Answers to the question as-

'Which game is liked the most'

'What percentage of students like kho-kho?'

'What percentage of students like kabaddi?'

can be obtained from the pie diagram at a glance.

Observe one more pie diagram.

fig 6.9

Figure 6.10 shows the annual financial planning of a school. From the pie diagram we see that

- 45% of the amount is reserved for educational equipment.

- 35% of the amount is shown for games.

- 10% of the amount is kept for sanitation.

- 10% of the amount is reserved for environment.
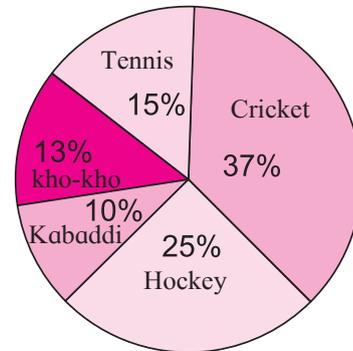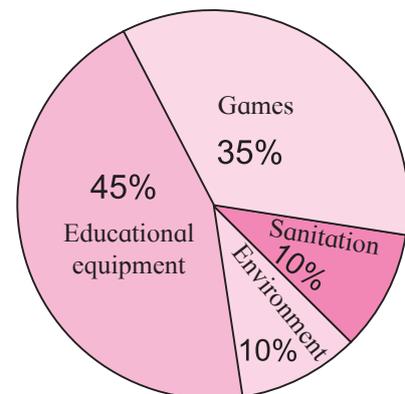
fig 6.10

In this way, we get information at a glance from a pie diagram.

Let us have more information about a pie diagram.

Many times we find information of different types in newspapers given in the form of pie diagrams. For example, the annual budget, performance of different nations in olympic games, etc.

Now we shall see, by examples, how to interpret the information from a pie diagram.

Example :

As deduced from a survey, the classification of skilled workers is shown in the pie diagram (fig 6.11). If the number of workers in the production sector is 4500, answer the following questions.
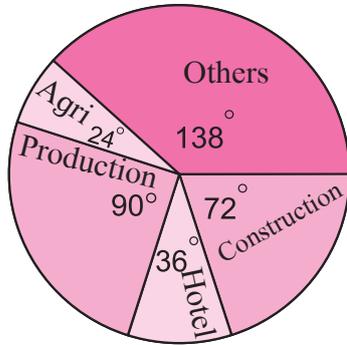

fig 6.11

(i) What is the total number of skilled workers in all fields?

(ii) What is the number of skilled workers in the field of constructions?

(iii) How many skilled workers are in agriculture?

(iv) Find the difference between the numbers of workers in the field of production and construction.

Solution : (i) Suppose, the total number of skilled workers in all fields is x.

∴ the central angle for x persons is = $360°$

Central angle for number of persons in production field

$$= \frac{\text{Number of persons in production field}}{x} \times 360$$

$$\therefore 90 = \frac{4500}{x} \times 360$$

$$\therefore x = 18000$$

∴ total number of skilled workers in all the fields together = 18000.

(ii) The angle shown for construction sector = $72°$.

$$\therefore 72 = \frac{\text{Number of persons in construction}}{18000} \times 360$$

∴ number of persons in construction field $= \frac{72 \times 18000}{360}$

$$= 3600$$

(iii) The central angle for agriculture field is $24°$.

$$\therefore 24 = \frac{\text{Number of workers in agriculture}}{\text{total skilled workers}} \times 360$$

$$24 = \frac{\text{Number of workers in agriculculture}}{18000} \times 360$$

∴ number of workers in agriculture $= \frac{24 \times 18000}{360}$

$$= 1200$$

(iv) The difference between angles relating fields of production and construction
$$= 90° − 72° = 18°.$$

∴ The difference between the central angles =

$$\frac{\text{Difference between numbres of workers in the fields}}{\text{Total number of skilled workers}} \times 360$$

$$18 = \frac{\text{Difference between the numbers of workers in the fields}}{18000} \times 360$$

Difference between the numbers of workers in the two fields $= \dfrac{18 \times 18000}{360}$

$$= 900$$

Let's remember!

- Every component of a data is shown by a sector associated with it.
- The measure of the central angle of the sector is in proportion with the number of scores in that component.
- The measure of central angle $(\theta) = \dfrac{\text{Number of scores in component}}{\text{Total number of scores}} \times 360^0$
- A circle of suitable radius should be drawn. Divide the circle in sectors such that the measure of central angle of each sector is proportional to the number of scores in its corresponding component in the data.

Let's learn.

To draw a Pie diagram

We have seen how to read a pie diagram. Now let us learn to draw it.

1. To draw a pie diagram, the whole circle is divided into sectors proportional to the components of the data

2. The measure of central angle of each sector is found by the following formula.

    The measure of central angle of sector θ

$$= \frac{\text{Number of scores in the components}}{\text{Total number of scores}} \times 360$$

A circle of a suitable radius is drawn. Then it is divided into sectors such that, the number of sectors is equal to the number of components in the data.

Let us understand the method through examples.

<div align="center">♫♫♫ Solved Examples ♫♫♫</div>

Ex. (1) In a bicycle shop, number of bicycles purchased and choice of their colours was as follows. Find the measures of sectors of a circle to show the information by a pie diagram.

Solution : In all 36 bicycles were purchased. Out of them 10 bicycles were white coloured.

∴ the measure of sector showing white coloured bicycles

$$= \frac{\text{Number of white bicycles}}{\text{Total number of bicycles}} \times 360$$

$$= \frac{10}{36} \times 360 = 100$$

The measures of angles of sector relating to bicycles of other colours can be calculated similarly which are shown in the adjacent table.

Colour	Number of bicycles	Central angle of the sector
White	10	$\frac{10}{36} \times 360° = 100°$
Black	9	$\frac{9}{36} \times 360° = 90°$
Blue	6	60°
Grey	7	70°
Red	4	40°
Total	36	360°

Ex. (2) The following table shows the daily supply of electricity to different places in a town. Show the information by a pie diagram.

Places	Factories	Houses	Roads	Shops	Offices	Others
Supply of electricity (Thousand units)	24	14	7	5	6	4

Solution : The total supply of electricity is 60,000 units. Let us find the measures of central angles and show in the table.

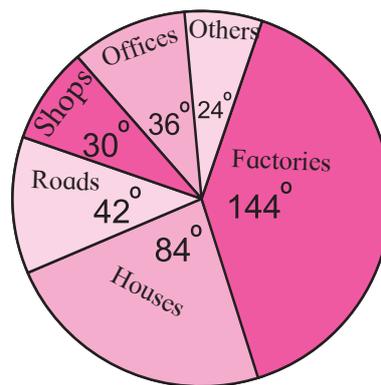| Supply of electricity	Unit	Measure of central angle
Factories	24	$\frac{24}{60} \times 360 = 144°$
Houses	14	$\frac{14}{60} \times 360 = 84°$
Roads	7	$\frac{7}{60} \times 360 = 42°$
Shops	5	$\frac{5}{60} \times 360 = 30°$
Offices	6	$\frac{6}{60} \times 360 = 36°$
Others	4	$\frac{4}{60} \times 360 = 24°$
Total	60	360°

fig 6.12

Steps of drawing pie chart :

(1) As shown in the figure, a circle and a radius is drawn. Then the sectors having measures of angles in the table, $(144°, 84°, 42°, 30°, 36°,$ and $24°)$ were drawn one by one, in the clockwise direction. (While drawing the sectors one by one, we can change their order.)

(2) The components of the data were recorded in the sectors.

Activity :

The monthly expenditure of a family on different items is shown in the following table. Calculate the related central angles and draw a pie chart.

Different items	Percentage of expenditure	Measure of central angle
Food	40	$\frac{40}{100} \times 360$ = ☐
Cloting	20	☐ × ☐ = ☐
House rent	15	☐ × ☐ = ☐
Education	20	☐ × ☐ = ☐
Expenditure	05	☐ × ☐ = ☐
Total	100	$360°$

Practice Set 6.6

1. The age group and number of persons, who donated blood in a blood donation camp is given below. Draw a pie diagram from it.

Age group (Yrs)	20–25	25–30	30–35	35–40
No. of persons	80	60	35	25

2. The marks obtained by a student in different subjects are shown. Draw a pie diagram showing the information.

Subject	English	Marathi	Science	Mathematics	Social science	Hindi
Marks	50	70	80	90	60	50

3. In a tree plantation programme, the number of trees planted by students of different classes is given in the following table. Draw a pie diagram showing the information.

Standard	5 th	6 th	7 th	8 th	9 th	10 th
No. of trees	40	50	75	50	70	75

4. The following table shows the percentages of demands for different fruits registered with a fruit vendor. Show the information by a pie diagram.

Fruits	Mango	Sweet lime	Apples	Cheeku	Oranges
Percentages of demand	30	15	25	20	10

5. The pie diagram in figure 6.13 shows the proportions of different workers in a town. Answer the following questions with its help.
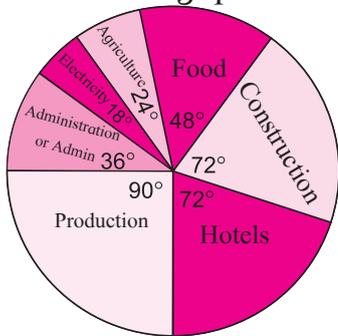

fig 6.13

(1) If the total workers is 10,000; how many of them are in the field of construction?

(2) How many workers are working in the administration?

(3) What is the percentage of workers in production?

6. The annual investments of a family are shown in the adjacent pie diagram. Answer the following questions based on it.

fig 6.14

(1) If the investment in shares is ₹ 2000/, find the total investment.

(2) How much amount is deposited in bank?

(3) How much more money is invested in immovable property than in mutual fund?
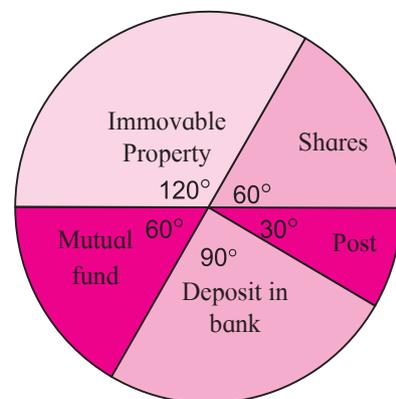
(4) How much amount is invested in post?

Miscellaneous Problems − 6

1. Find the correct answer from the alternatives given.

(1) The persons of O− blood group are 40%. The classification of persons based on blood groups is to be shown by a pie diagram. What should be the measures of angle for the persons of O− blood group?

(A) 114°　　　　(B) 140°　　　　(C) 104°　　　　(D) 144°

(2) Different expenditures incurred on the construction of a building were shown by a pie diagram. The expenditure ₹ 45,000 on cement was shown by a sector of central angle of 75°. What was the total expenditure of the construction ?

(A) 2,16,000   (B) 3,60,000   (C) 4,50,000   (D) 7,50,000

(3) Cumulative frequencies in a grouped frequency table are useful to find . . .

(A) Mean   (B) Median   (C) Mode   (D) All of these

(4) The formula to find mean from a grouped frequency table is $\overline{X} = A + \dfrac{\sum f_i u_i}{\sum f_i} \times h$ ॰ In the formula $u_i = \ldots$

(A) $\dfrac{x_i + A}{g}$ (B) $(x_i - A)$ (C) $\dfrac{x_i - A}{g}$ (D) $\dfrac{A - x_i}{g}$

(5)

Distance Covered per litre (km)	12−14	14−16	16−18	18−20
No. of cars	11	12	20	7

The median of the distances covered per litre shown in the above data is in the group . . . . . .

(A) 12−14   (B) 14−16   (C) 16−18   (D) 18−20

(6)

No. of trees planted by each student	1−3	4−6	7−9	10−12
No. of students	7	8	6	4

The above data is to be shown by a frequency polygon. The coordinates of the points to show number of students in the class 4−6 are . . . .

(A) (4, 8)   (B) (3, 5)   (C) (5, 8)   (D) (8, 4)

2. The following table shows the income of farmers in a grape season. Find the mean of their income.

Income (Thousand Rupees)	20−30	30−40	40−50	50−60	60−70	70−80
Farmers	10	11	15	16	18	14

3. The loans sanctioned by a bank for construction of farm ponds are shown in the following table. Find the mean of the loans.

Loan (Thousand rupees)	40−50	50−60	60−70	70−80	80−90
No. of farm ponds	13	20	24	36	7

4. The weekly wages of 120 workers in a factory are shown in the following frequency distribution table. Find the mean of the weekly wages.

Weekly wages (Rupees)	0-2000	2000-4000	4000-6000	6000-8000
No. of workers	15	35	50	20

5. The following frequency distribution table shows the amount of aid given to 50 flood affected families. Find the mean of the amount of aid.

Amount of aid (Thosand rupees)	50-60	60-70	70-80	80-90	90-100
No. of families	7	13	20	6	4

6. The distances covered by 250 public transport buses in a day is shown in the following frequency distribution table. Find the median of the distances.

Distance (km)	200-210	210-220	220-230	230-240	240-250
No. of buses	40	60	80	50	20

7. The prices of different articles and demand for them is shown in the following frequency distribution table. Find the median of the prices.

Price (Rupees)	20 less than	20-40	40-60	60-80	80-100
No. of articles	140	100	80	60	20

8. The following frequency table shows the demand for a sweet and the number of customers. Find the mode of demand of sweet.

Weight of sweet (gram)	0-250	250-500	500-750	750-1000	1000-1250
No. of customers	10	60	25	20	15

9. Draw a histogram for the following frequency distribution.

Use of electricity (Unit)	50-70	70-90	90-110	110-130	130-150	150-170
No. of families	150	400	460	540	600	350

10. In a handloom factory different workers take different periods of time to weave a saree. The number of workers and their required periods are given below. Present the information by a frequency polygon.

No. of days	8–10	10–12	12–14	14–16	16–18	18–20
No. of workers	5	16	30	40	35	14

11. The time required for students to do a science experiment and the number of students is shown in the following grouped frequency distribution table. Show the information by a histogram and also by a frequency polygon.

Time required for experiment (minutes)	20–22	22–24	24–26	26–28	28–30	30–32
No. of students	8	16	22	18	14	12

12. Draw a frequency polygon for the following grouped frequency distribution table.

Age of the donor (Yrs.)	20–24	25–29	30–34	35–39	40–44	45–49
No. of blood doners	38	46	35	24	15	12

13. The following table shows the average rainfall in 150 towns. Show the information by a frequency polygon.

Average rainfall (cm)	0–20	20–40	40–60	60–80	80–100
No. of towns	14	12	36	48	40

14. Observe the adjacent pie diagram. It shows the percentages of number of vehicles passing a signal in a town between 8 am and 10 am
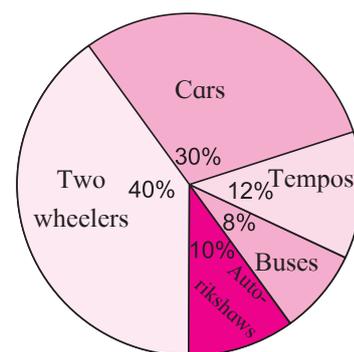
   (1) Find the central angle for each type of vehicle.

   (2) If the number of two-wheelers is 1200, find the number of all vehicles.

fig 6.15

15. The following table shows causes of noise pollution. Show it by a pie diagram.

Construction	Traffic	Aircraft take offs	Industry	Trains
10%	50%	9%	20%	11%

16. A survey of students was made to know which game they like. The data obtained in the survey is presented in the adjacent pie diagram. If the total number of students are 1000,
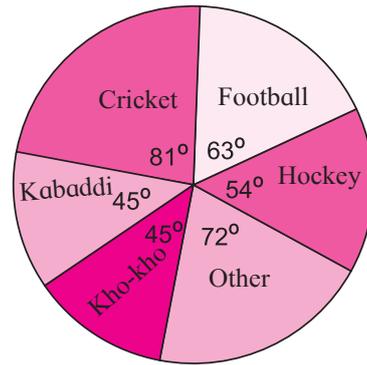

fig 6.16

   (1) How many students like cricket?

   (2) How many students like football?

   (3) How many students prefer other games?

17. Medical check up of 180 women was conducted in a health centre in a village. 50 of them were short of haemoglobin, 10 suffered from cataract and 25 had respiratory disorders. The remaining women were healthy. Show the information by a pie diagram.

18. On an environment day, students in a school planted 120 trees under plantation project. The information regarding the project is shown in the following table. Show it by a pie diagram.

Tree name	Karanj	Behada	Arjun	Bakul	Kadunimb
No. of trees	20	28	24	22	26

Answers

1. Linear Equations In Two Variables

Practice Set 1.1

2. (1) (2, 4) (2) (3, 1) (3) (6,1) (4) (5, 2)

(5) (-1, 1) (6) (1, 3) (7) (3, 2) (8) (7, 3)

Practice Set 1.2

1. (1)

x	3	−2	0
y	0	5	3
(x, y)	(3, 0)	(−2, 5)	(0, 3)

(2)

x	4	−1	0
y	0	−5	−4
(x, y)	(4, 0)	(−1,−5)	(0,−4)

2. (1) (5, 1) (2) (4, 1) (3) (3, -3) (4) (-1, -5) (5) (1, 2.5) (6) (8, 4)

Practice Set 1.3

1. $\begin{vmatrix} 3 & 2 \\ 4 & 5 \end{vmatrix} = 3 \times \boxed{5} - \boxed{2} \times 4 = \boxed{15} - 8 = \boxed{7}$

2. (1) −18 (2) 21 (3) $-\dfrac{4}{3}$

3. (1) (2, −1) (2) (−2, 4) (3) (3, −2) (4) (2, 6) (5) (6, 5) (6) $(\dfrac{5}{8}, \dfrac{1}{4})$

Practice Set 1.4

1. (1) $(\dfrac{1}{9}, 1)$ (2) (3, 2) (3) $(\dfrac{5}{2}, -2)$ (4) (1, 1)

Practice Set 1.5

1. The numbers are 5 and 2 **2.** $x = 12$, $y = 8$, Area = 640 sq. unit,

Perimeter = 112 unit **3.** Son's age is 15 years, father's age is 40 years

4. $\dfrac{7}{18}$ **5.** A − 30 kg, B − 55 kg **6.** 150 km.

Problem Set 1

1. (1) B (2) A (3) D (4) C (5) A

2.

x	−5	$\dfrac{3}{2}$
y	$-\dfrac{13}{6}$	0
(x, y)	$(-5, -\dfrac{13}{6})$	$(\dfrac{3}{2}, 0)$

3. (1) (3, 2) (2) (-2, -1)   (3) (0, 5)   (4) (2, 4)   (5) (3, 1)

4. (1) 22   (2) −1   (3) 13

5. (1) $(-\frac{2}{3}, 2)$ (2) (1, 4)   (3) $(\frac{1}{2}, -\frac{1}{2})$ (4) $(\frac{7}{11}, \frac{116}{33})$ (5) (2, 6)

6. (1) (6, −4)   (2) $(-\frac{1}{4}, -1)$ (3) (1, 2)   (4) (1, 1)   (5) (2, 1)

7. (2) Tea; ₹300 per kg.
    sugar; ₹ 40 per kg.

(3) ₹100 notes 20
    ₹50 notes 10

(4) Manisha's age 23 years
    Savita's age 8 years.

(5) Skilled worker's wages ₹ 450.
    unskilled worker's wages ₹ 270.

(6) Hamid's speed 50 km/hr.
    Joseph's speed 40 km/hr.

2. Quadratic Equations

Practice Set 2.1

1. Any equations of the type $m^2 + 5m + 3 = 0$, $y^2 - 3 = 0$

2. (1), (2), (4), (5) are quadratic equations.

3. (1) $y^2 + 2y - 10 = 0$, $a = 1, b = 2, c = -10$

(2) $x^2 - 4x - 2 = 0$, $a = 1, b = -4, c = -2$

(3) $x^2 + 4x + 3 = 0$, $a = 1, b = 4, c = 3$

(4) $m^2 + 0m + 9 = 0$, $a = 1, b = 0, c = 9$

(5) $6p^2 + 3p + 5 = 0$, $a = 6, b = 3, c = 5$

(6) $x^2 + 0x - 22 = 0$, $a = 1, b = 0, c = -22$

4. (1) 1 is a root, −1 is not.   (2) $\frac{5}{2}$ is a root, 2 is not.

5. k = 3   **6.** k = −7

Practice Set 2.2

1. (1) 9, 6   (2) −5, 4   (3) $-13, -\frac{1}{2}$ (4) $5, -\frac{3}{5}$

(5) $\frac{1}{2}, \frac{1}{2}$ (6) $\frac{2}{3}, -\frac{1}{2}$ (7) $-\frac{5}{\sqrt{2}}, -\sqrt{2}$ (8) $\frac{\sqrt{2}}{\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}}$

(9) 25, −1   (10) $-\frac{3}{5}, \frac{3}{5}$ (11) 0, 3   (12) $-\sqrt{11}, \sqrt{11}$

Practice Set 2.3

1. (1) $4, -5$ (2) $(\sqrt{6}-1), (-\sqrt{6}-1)$ (3) $\dfrac{\sqrt{13}+5}{2}, \dfrac{-\sqrt{13}+5}{2}$

(4) $\dfrac{\sqrt{2}+2}{3}, \dfrac{-\sqrt{2}+2}{3}$ (5) $-\dfrac{5}{4}, -\dfrac{5}{2}$ (6) $\dfrac{2+\sqrt{39}}{5}, \dfrac{2-\sqrt{39}}{5}$

Practice Set 2.4

1. (1) $1, -7, 5$ (2) $2, -5, 5$ (3) $1, -7, 0$

2. (1) $-1, -5$ (2) $\dfrac{3+\sqrt{17}}{2}, \dfrac{3-\sqrt{17}}{2}$ (3) $\dfrac{-1+\sqrt{22}}{3}, \dfrac{-1-\sqrt{22}}{3}$

(4) $\dfrac{2+\sqrt{14}}{5}, \dfrac{2-\sqrt{14}}{5}$ (5) $\dfrac{-1+\sqrt{73}}{6}, \dfrac{-1-\sqrt{73}}{6}$ (6) $-1, -\dfrac{8}{5}$

3. $-\sqrt{3}, -\sqrt{3}$

Practice Set 2.5

1. (1) Roots are distinct and real when $b^2 - 4ac = 5$, not real when $b^2 - 4ac = -5$.

(2) $x^2 + 7x + 5 = 0$ (3) $\alpha + \beta = 2, \alpha \times \beta = -\dfrac{3}{2}$

2. (1) 53 (2) -55 (3) 0

3. (1) Real and equal. (2) Real and unequal. (3) Not real.

4. (1) $x^2 - 4x = 0$ (2) $x^2 + 7x - 30 = 0$

(3) $x^2 - \dfrac{1}{4} = 0$ (4) $x^2 - 4x - 1 = 0$

5. $k = 3$ 6. (1) 18 (2) 50

7. (1) $k = 12$ or $k = -12$ (2) $k = 6$

Practice Set 2.6

1. 9 years 2. 10 and 12 3. In vertical row 10, in horizontal row 15.

4. Kishor's present age is 10 years and Vivek's present age is 15 years

5. 10 marks 6. No. of pots 6, production cost of each is ₹ 100.

7. 6 km/hr 8. For Nishu 6 days, for Pintu 12 days.

9. Divisor = 9, quotient = 51 10. AB = 7 cm, CD = 15 cm, AD = BC = 5 cm.

Problem Set 2

1. (1) B (2) A (3) C (4) B (5) B (6) D (7) C (8) C

2. (1) and (3) are quadratic equations.

3. (1) −15 (2) 1 (3) 21

4. k = 3   5. (1) $x^2 - 100 = 0$ (2) $x^2 - 2x - 44 = 0$ (3) $x^2 - 7x = 0$

6. (1) Not real.   (2) Real and unequal   (3) Real and equal

7. (1) $\dfrac{1+\sqrt{21}}{2}, \dfrac{1-\sqrt{21}}{2}$ (2) $\dfrac{1}{2}, -\dfrac{1}{5}$ (3) 1, −4

   (4) $\dfrac{-5+\sqrt{5}}{2}, \dfrac{-5-\sqrt{5}}{2}$ (5) Roots are not real. (6) $(2 + \sqrt{7}), (2 - \sqrt{7})$

8. $m = 14$ 9. $x^2 - 5x + 6 = 0$ 10. $x^2 - 4pqx - (p^2 - q^2)^2 = 0$

11. ₹ 100 with Sagar, ₹ 150 with Mukund.

12. 12 and $\sqrt{24}$ or 12 and $-\sqrt{24}$ 13. No. of students 60

14. Breadth 45 m. length 100 m, side of the pond 15 m.

15. For larger tap 3 hours and for smaller tap 6 hours.

3. Arithmetic Progression

Practice Set 3.1

1. (1) Yes, $d = 2$ (2) Yes, $d = \dfrac{1}{2}$ (3) Yes, $d = 4$ (4) No

   (5) Yes, $d = -4$ (6) Yes, $d = 0$ (7) Yes, $d = \sqrt{2}$ (8) Yes, $d = 5$

2. (1) 10, 15, 20, 25, . . . (2) −3, −3, −3, −3, . . . (3) −7, −6.5, −6, −5.5, . .

   (4) −1.25, 1.75, 4.75, 7.75, . . .(5) 6, 3, 0, −3 . . .   (6) −19, −23, −27, −31

3. (1) $a = 5, d = -4$ (2) $a = 0.6, d = 0.3$ (3) $a = 127, d = 8$ (4) $a = \dfrac{1}{4}, d = \dfrac{1}{2}$

Practice Set 3.2

1. (1) $d = 7$ (2) $d = 3$ (3) $a = -3, d = -5$ (4) $a = 70, d = -10$

2. Yes. 121   3. 104   4. 1115   5. -121   6. 180

7. 55   8. 55ᵗʰ   9. 60   10. 1

Practice Set 3.3

1. 1215   2. 15252   3. 30450   5. 5040

5. 2380   6. 60   7. 4, 9, 14 or 14, 9, 4   8. −3, 1, 5, 9

Practice Set 3.4

1. ₹ 70455   2. First instalment ₹ 1000, last instalment ₹ 560.   3. ₹ 1,92,000

4. 48, 1242   5. −20°, −25°,−30°,−35°,−40°,−45°   6. 325

Problem Set 3

1. (1) B (2) C (3) B (4) D (5) B (6) C (7) C (8) A (9) A (10) B

2. 40   3. 1, 6, 11, . . .   4. −195   5. 16, -21   6. -1   7. 6, 10

8. 8   9. 67, 69, 71   10. 3, 7, 11, ... , 147   14. ₹ 2000.

4. Financial Planning

Practice Set 4.1

1. CGST 6%, SGST 6%   **2.** SGST 9%, GST 18%
3. CGST ₹ 784 and SGST ₹ 784
4. The customer gets the belt for ₹ 691.48.
5. Taxable value of toy car is ₹ 1500, CGST ₹ 135, SGST ₹ 135
6. (1) Rate of SGST 14%   (2) Rate of GST on AC 28%
    (3) Taxable value of AC ₹ 40,000.   (4) Total GST ₹ 11,200.
    (5) CGST ₹ 5600.   (6) SGST ₹ 5600.
7. Prasad gets the washing machine for ₹ 48,640 and CGST ₹ 5320, SGST ₹ 5320.

Practice Set 4.2

1. Payable GST ₹ 22,000.
2. Input Tax Credit for Nazama is ₹ 12,500 and her payable GST is ₹ 2250.
3. Ameer Enterprises : Payable GST ₹ 300, payable CGST ₹ 150,
    payable SGST ₹ 150.
    Akabari Brothers : payable GST ₹ 400, payable CGST ₹ 200,
    payable SGST ₹ 200.
4. Payable GST ₹ 100 so CGST ₹ 50 and UTGST ₹ 50. **5.** CGST = SGST = ₹ 900

Practice Set 4.3

1. (1) MV ₹ 100   (2) FV ₹ 75   (3) At discount of ₹ 5.
2. 25%   **3.** ₹ 37,040   **4.** 800 shares
5. Rate of return 5.83%   **6.** Company A– more profitable.

Practice Set 4.4

1. ₹ 200.60   **2.** ₹ 999
3.

No. of shares	MV of shares	Total value	Brokerage 0.2%	9% CGST on brokerage	9% SGST on brokerage	Total value of shares
100 B	₹ 45	₹ 4500	₹ 9	₹ 0.81	₹ 0.81	₹ 4510.62
75 S	₹ 200	₹15000	₹ 30	₹ 2.70	₹ 2.70	₹ 14964.60

4. No. of shares sold = 100.   **5.** Loss of ₹ 8560.

Problem Set 4A

1. (1) C (2) B (3) D (4) B (5) A (6) B
2. Total bill ₹ 28,800 , CGST ₹ 3150, SGST ₹ 3150.

3. ₹ 997.50   4. ₹ 12,500   5. ITC ₹ 4250, payable tax ₹ 250

6. ITC ₹ 1550, payable CGST ₹ 5030, payable SGST ₹ 5030.

7. Taxable value ₹ 75,000, CGST ₹ 4500, SGST ₹ 4500

8. (1) Wholesaler's tax invoice : CGST ₹ 16200; SGST ₹ 16200.

      Retailer's tax invoice: CGST ₹ 19,800; SGST ₹ 19,800.

     (2) Wholesaler : payable CGST ₹ 2700 and payable SGST ₹ 2700,

      Retailer : payable CGST ₹ 3600 and payable SGST ₹ 3600

9. (1) Anna Patil's invoice : CGST ₹ 1960, SGST ₹ 1960

     (2) Trader in Vasai : CGST ₹ 2352 and SGST ₹ 2352

     (3) Trader in Vasai : payable CGST ₹ 392 and payable SGST ₹ 392

10.

     (1)

Person	Payable CGST (₹)	Payable SGST (₹)	Payable GST (₹)
Manufacturer	300	300	600
Distributor	360−300 = 60	60	120
Retailer	390−360 = 30	30	60
Total Tax	390	390	780

     (2) Finally, the customer will get the article for ₹ 7280.

     (3) Manufacturer to distributor B2B, distributor to retailer B2B,

        retailer to customer B2C

Problem Set 4B

1. (1) B   (2) B   (3) A   (4) C   (5) A

2. ₹ 130.39   3. 22.2%   4. will get ₹ 21,000 .

5. Will get 500 shares.   6. Profit ₹ 1058.52   7. Company B, as returns are more

8. Will get 1000 shares. 9. ₹ 118.

10. (1) ₹ 1,20,000   (2) ₹ 360   (3) ₹ 64.80 (4) ₹ 120424.80.

11.   1% profit

5. Probability

Practice Set 5.1

1.   (1) 8 (2) 7 (3) 52 (4) 11

Practice Set 5.2

1.   (1) S = {1H, 1T, 2H, 2T, 3H, 3T, 4H, 4T, 5H, 5T, 6H, 6T}   n(S) = 12

(2) S = {23, 25, 32, 35, 52, 53}   n(S) = 6

2.   S = {Red, Purple, Orange, Yellow, Blue, Green}   n(S) = 6

3.   S = {Tuesday, Sunday, Friday, Wednesday, Monday, Saturday}   n(S) = 6

4.   (1) $\boxed{B_1B_2}$ (2) $\boxed{G_1G_2}$ (3) $\boxed{B_1G_1}$ $\boxed{B_2G_1}$ $\boxed{B_1G_2}$ $\boxed{B_2G_2}$

(4) S = {B_1B_2, B_1G_1, B_1G_2, B_2G_1, B_2G_2, G_1G_2}

Practice Set 5.3

1.   (1) S = {1, 2, 3, 4, 5, 6}   n(S) = 6

A = {2, 4, 6} n(A) = 3,   B = {1, 3, 5} n(B) = 3,   C = {2, 3, 5}   n(C) = 3

(2) S = {(1,1), . . ., (1, 6), (2,1), . . ., (2, 6), (3, 1), . . .,(3, 6),

(4, 1), . . .,(4,6),(5, 1), . . .,(5, 6),(6, 1), . . .,(6, 6)}   n(S) = 36

A = {(1, 5) (2, 4) (3, 3) (4, 2) (5, 1) (6, 6)}   n(A) = 6

B = {(4, 6) (5, 5) (5, 6) (6, 4) (6, 5) (6, 6)}   n(B) = 6

C = {(1, 1) (2, 2) (3, 3) (4, 4) (5, 5) (6, 6)}   n(C) = 6

(3) S = {HHH, HHT, HTT, HTH, THT, TTH, THH, TTT} n(S) = 8

A = {HHH, HHT, HTH, THH}   n(A) = 4

B = {TTT}   n(B) = 1

C = {HHH, HHT, THH}   n(C) = 3

(4) S = {10, 12, 13, 14, 15, 20, 21, 23, 24, 25, 30, 31, 32, 34, 35, 40, 41, 42, 43,

45, 50, 51, 52, 53, 54}   n(S) = 25

A = {10, 12, 14, 20, 24, 30, 32, 34, 40, 42, 50, 52, 54}   n(A) = 13

B = {12, 15, 21, 24, 30, 42, 45, 51, 54}   n(B) = 9

C = {51, 52, 53, 54}   n(C) = 4

(5) S = {M_1M_2, M_1M_3, M_1F_1, M_1F_2, M_2M_3, M_2F_1, M_2F_2, M_3F_1, M_3F_2, F_1F_2}

n(S) = 10

A = {M_1F_1, M_1F_2, M_2F_1, M_2F_2, M_3F_1, M_3F_2, F_1F_2}   n(A) = 7

B = {M_1F_1, M_1F_2, M_2F_1, M_2F_2, M_3F_1, M_3F_2}   n(B) = 6

C = {M_1M_2, M_1M_3, M_2M_3}   n(C) = 3

(6) S = {H1, H2, H3, H4, H5, H6 T1, T2, T3, T4, T5, T6}   n(S) = 12

A = {H1, H3, H5}   n(A) = 3

B = {H2, H4, H6, T2, T4, T6}   n(B) = 6

C = { }   n(C) = 0

Practice Set 5.4

1.   (1) $\frac{3}{4}$, (2) $\frac{1}{4}$ **2.** (1) $\frac{1}{6}$ (2) 0   (3) $\frac{5}{12}$

3. (1) $\frac{7}{15}$ (2) $\frac{1}{5}$ 4. (1) $\frac{4}{5}$ (2) $\frac{1}{5}$ 5. (1) $\frac{1}{13}$ (2) $\frac{1}{4}$

Problem Set – 5

1. (1) B (2) B (3) C (4) A (5) A   2. Vasim's   3. (1) $\frac{1}{11}$ (2) $\frac{6}{11}$

4. $\frac{5}{26}$ 5. (1) $\frac{4}{9}$ (2) $\frac{1}{3}$ (3) $\frac{4}{9}$ 6. $\frac{1}{2}$ 7. (1) $\frac{1}{3}$ (2) $\frac{1}{6}$

8. (1) $\frac{1}{2}$ (2) $\frac{1}{6}$ 9. $\frac{1}{25}$ 10. (1) $\frac{1}{8}$ (2) $\frac{1}{2}$ (3) $\frac{3}{4}$ (4) 1

11. (1) $\frac{5}{6}$ (2) $\frac{1}{6}$ (3) 1 (4) 0   12. (1) $\frac{1}{3}$ (2) $\frac{2}{3}$ (3) $\frac{2}{3}$ 13. $\frac{2}{11}$

14. $\frac{13}{40}$ 15. (1) $\frac{3}{10}$ (2) $\frac{3}{10}$ (3) $\frac{1}{5}$ 16. $\frac{11}{36}$

6. Statistics

Practice Set 6.1

(1) 4.36 hrs   (2) ₹ 521.43.   (3) 2.82 litre   (4) ₹ 35310

(5) ₹ 985   or   ₹ 987.5.   (6) ₹ 3070   or   ₹ 3066.67.

Practice Set 6.2

(1) 11.4 hrs (2) 184.4 means 184 mangoes approximately (3) 74.558 ≈ 75 vehicles
(4) 52750 lamps

Practice Set 6.3

1. 4.33 litre   2. 72 unit   3. 9.94 litre   4. 12.31 years

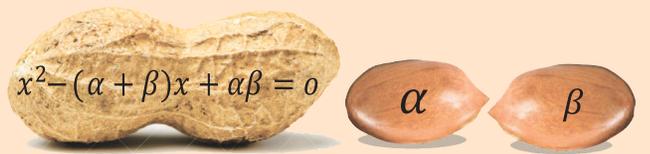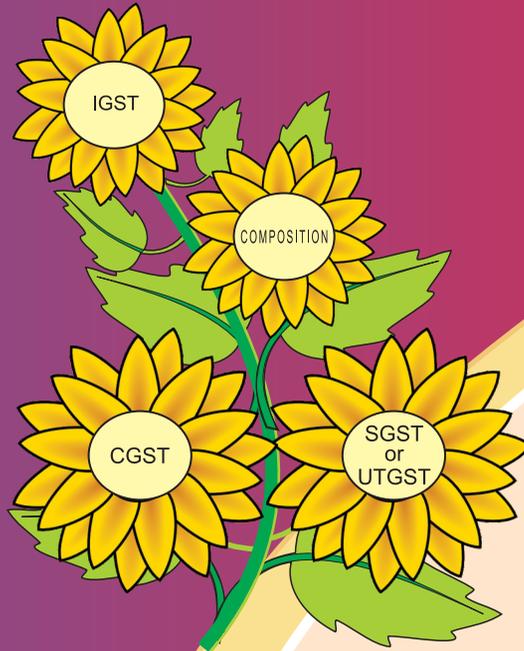### Practice Set 6.5

1. (1) 60–70   (2) 20–30 and 90–100   (3) 55   (4) 80 and 90   (5) 15

Practice Set 6.6

5. (1) 2000 (2) 1000 (3) 25%

6. (1) ₹ 12000 (2) ₹ 3000 (3) ₹ 2000 (4) ₹ 1000.

Problem Set – 6

1. (1) D   (2) A   (3) B   (4) C   (5) C   (6) C

2. ₹ 52,500   3. ₹ 65,400   4. ₹ 4250

5. ₹ 72,400   6. 223.13 km.   7. ₹ 32   8. 397.06 gm

14. (1) Cars – 108°, Tempos – 43°, Buses – 29°, Auto–rickshaws – 36°,
    Two wheelers – 144°

    (2) Total number of vehicles – 3000

16. (1) Cricket – 225, (2) Football – 175 (3) Other games– 200.

IGST

COMPOSITION

CGST

SGST
or
UTGST

$$x^2 - (\alpha + \beta)x + \alpha\beta = o$$

α β